Higher

Mathematics

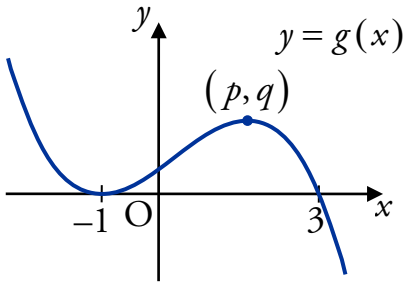
Graphs

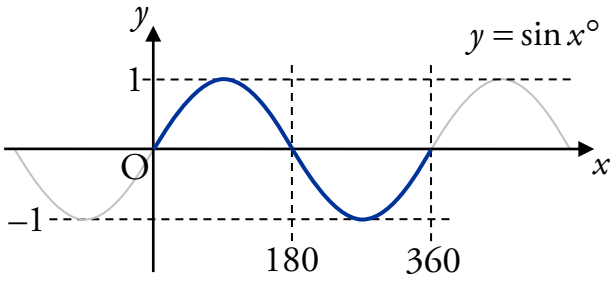
Contents

CfE Edition

8 Graph Transformations EF

The graphs below represent two functions. One is a cubic and the other is a sine wave, focusing on the region between 0 and 360.





In the following pages we will see the effects of three different "transformations" on these graphs: translation, reflection and scaling.

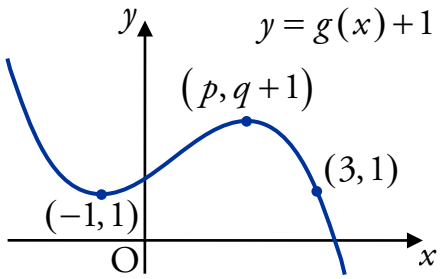
Translation

A **translation** moves every point on a graph a fixed distance in the same direction. The shape of the graph does not change.

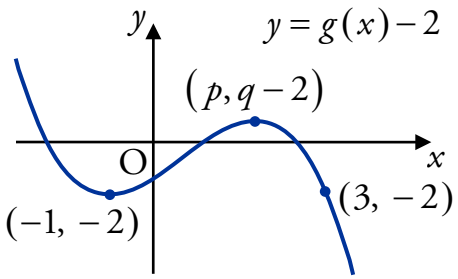
Translation parallel to the y-axis

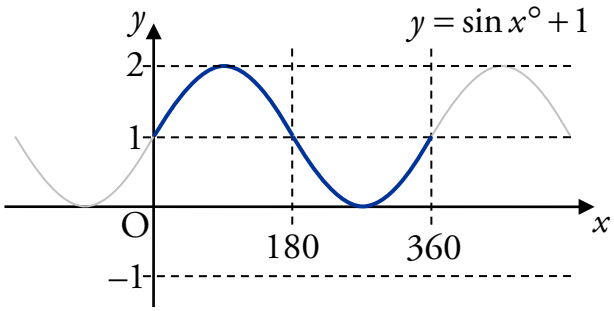
$f(x)+a$ moves the graph of $f(x)$ up or down. The graph is moved up if a is positive, and down if a is negative.

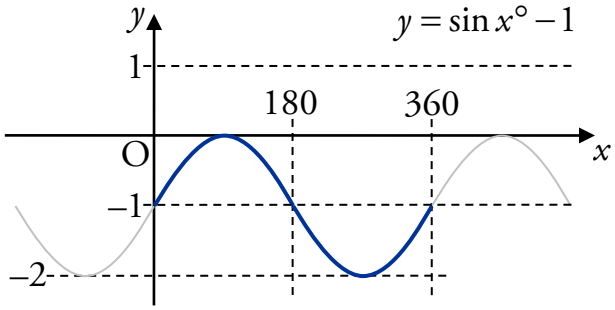
a is positive



a is negative



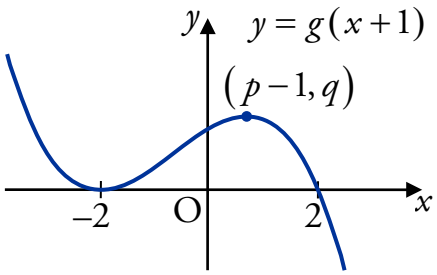




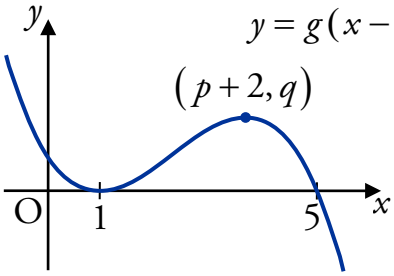
Translation parallel to the x-axis

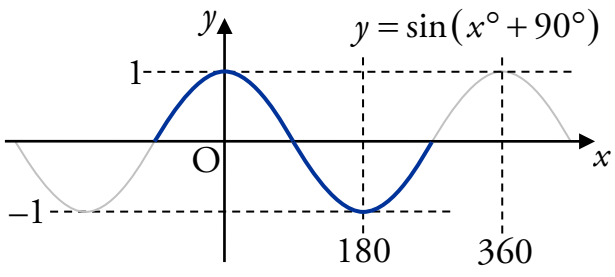
$f(x+a)$ moves the graph of $f(x)$ left or right. The graph is moved left if a is positive, and right if a is negative.

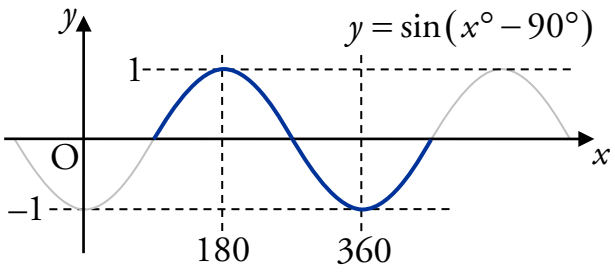
a is positive



a is negative







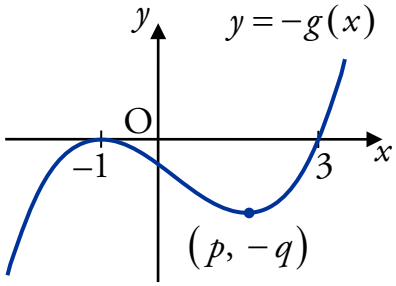
Reflection

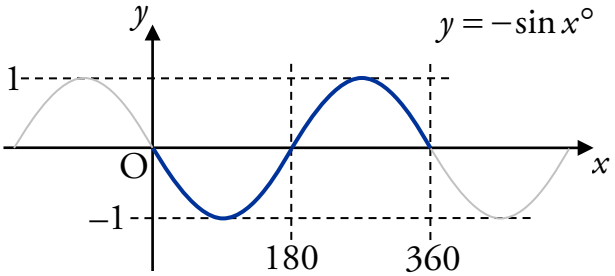
A **reflection** flips the graph about one of the axes.

When reflecting, the graph is flipped about one of the axes. It is important to apply this transformation *before* any translation.

Reflection in the x-axis

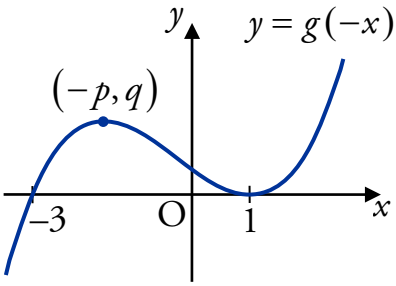
$-f(x)$ reflects the graph of $f(x)$ in the x-axis.

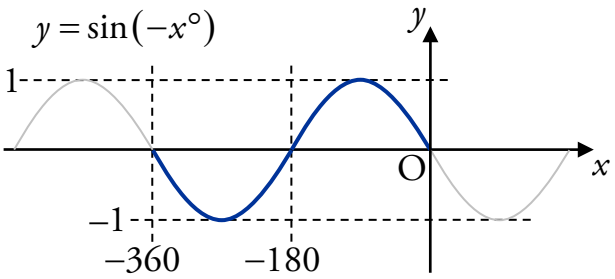




Reflection in the y-axis

$f(-x)$ reflects the graph of $f(x)$ in the y-axis.





From the graphs, $\sin(-x)° = -\sin x°$

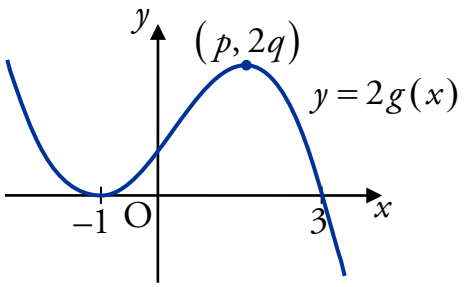
Scaling

A **scaling** stretches or compresses the graph along one of the axes.

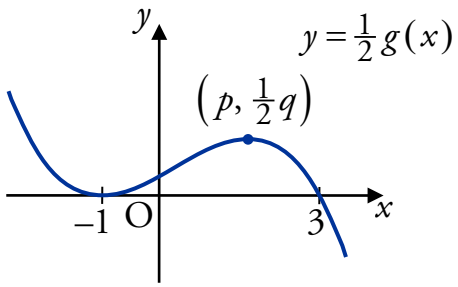
Scaling vertically

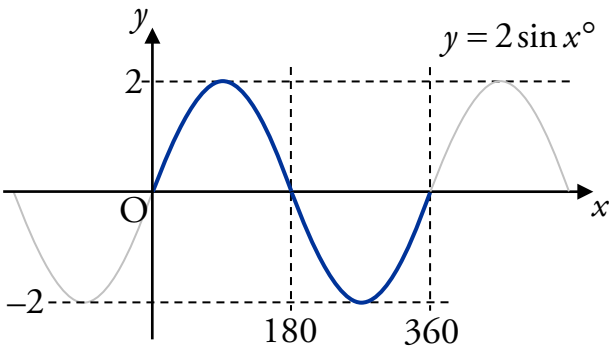
$kf(x)$ scales the graph of $f(x)$ in the vertical direction. The y-coordinate of each point on the graph is multiplied by k, roots are unaffected. These examples consider positive k.

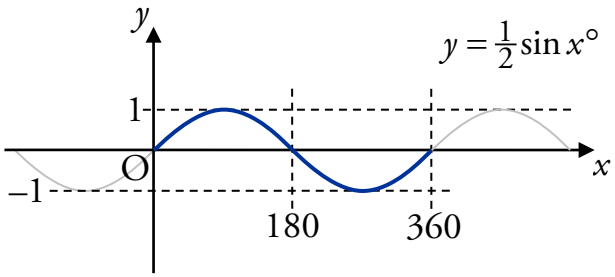
$k > 1$ stretches



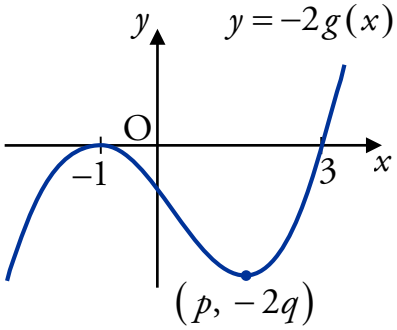
$0 < k < 1$ compresses







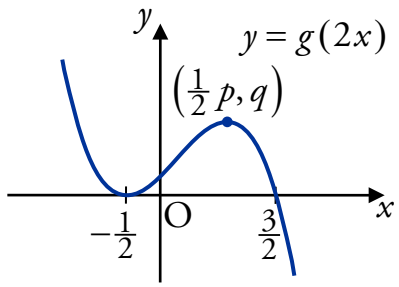
Negative k causes the same scaling, but the graph must then be reflected in the x-axis:



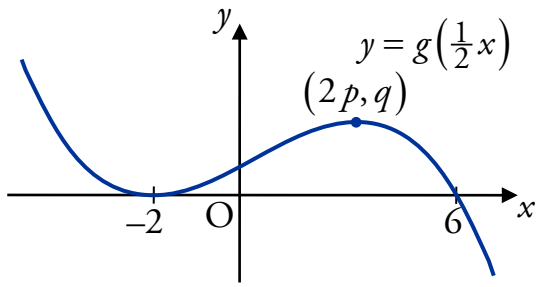
Scaling horizontally

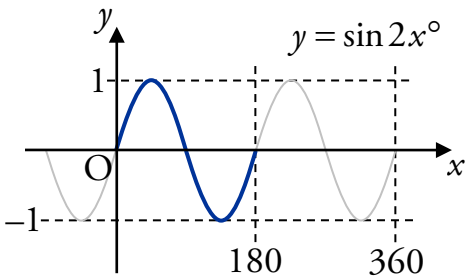
$f(kx)$ scales the graph of $f(x)$ in the horizontal direction. The coordinates of the y-axis intercept stay the same. The examples below consider positive k.

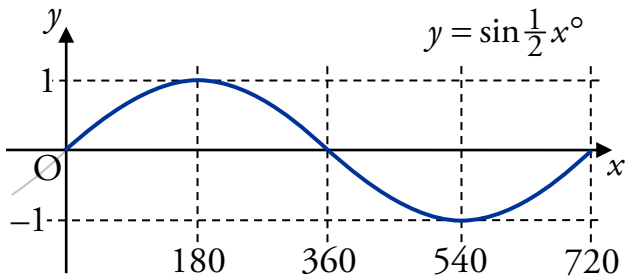
$k > 1$



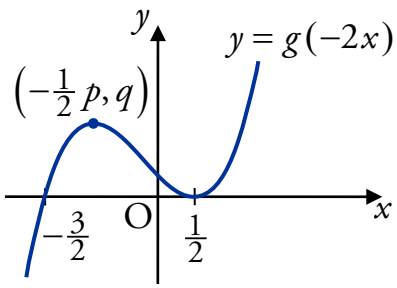
$0 < k < 1$





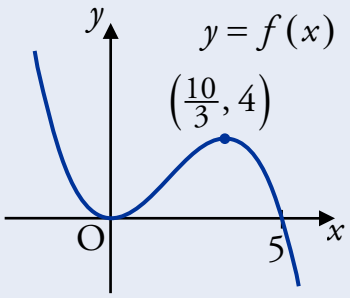


Negative k causes the same scaling, but the graph must then be reflected in the y-axis:



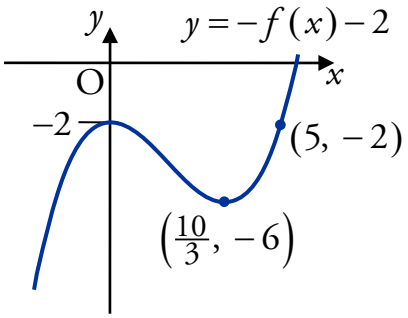
EXAMPLES

1. The graph of $y = f(x)$ is shown below.

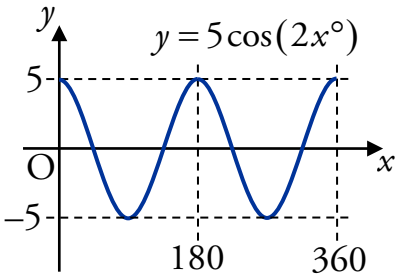


Sketch the graph of $y = -f(x) - 2$.

Reflect in the x-axis, then shift down by 2:



2. Sketch the graph of $y = 5\cos(2x°)$ where $0 \le x \le 360$.



Remember

The graph of $y = \cos x$: