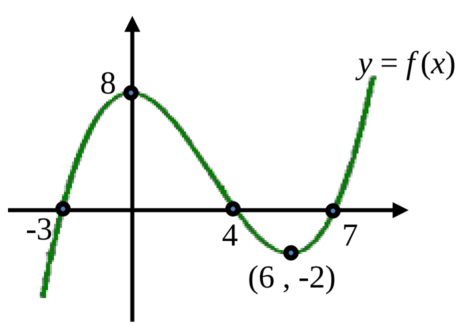
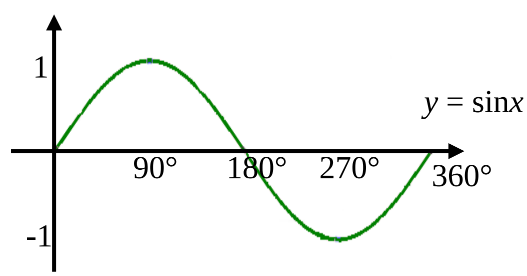
Graphs of Functions

Transformations:

- There are 3 types of **TRANSFORMATIONS** which can be applied to any Graph:.
  - A **TRANSLATION** moves every point on a graph the same amount in a specific direction.
  - A **REFLECTION** "flips" the graph obout one of the axis.
  - A **SCALING** stretches or compresses the graph in a specific direction.
- Each transformation can be applied vertically or horizontally depending where it's stated in the function:
  - If the transformation is **INSIDE** the bracket it is a **HORIZONTAL** movement and affects the graph in the **OPPOSITE** manner than first thought!!!!
  - If the transformation is **OUTSIDE** the bracket it is a **VERTICAL** movement and affects the graph in the normal way!!!
- We will now look at **ALL** these transformations in detail using the following 2 graphs:



Cubic Function



Sine Function

Translations:

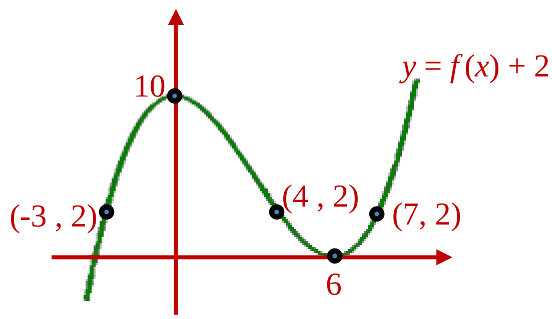
- There are 2 types of **TRANSLATIONS** which can be applied to a Graph:.
  - ➢ Horizontally: $f(x + a)$ This will cause the graph to move parallel to the x-axis; to the **LEFT** if $\boldsymbol{a} > 0$ and **RIGHT** if $\boldsymbol{a} < 0$ ← **Opposite from the way you thimk!!**
    Only the x-coordinate will change
  - ➢ Vertically: $f(x) + a$ This will cause the graph to move parallel to the y-axis; **UP** if $\boldsymbol{a} > 0$ and **DOWN** if $\boldsymbol{a} < 0$
    Only the y-coordinate will change

Examples:

1. Sketch the following graphs:

a) $y = f(x) + 2$

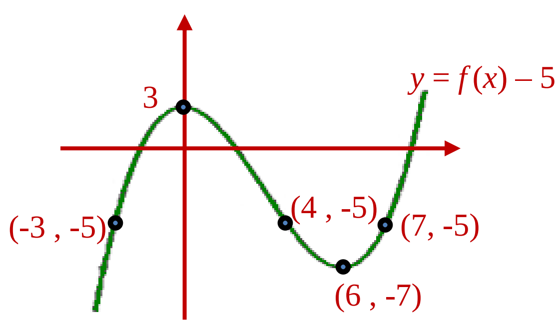
Shift the Graph **UP** 2



You must annotate the graph by marking in ALL the given points on the new graph to gain full marks!!!

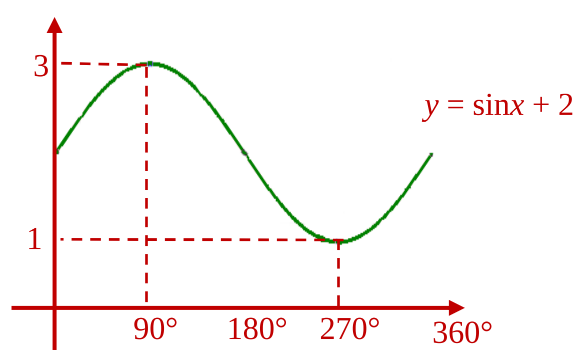
b) $y = f(x) - 5$

Shift the Graph **DOWN** 5



c) $y = \sin x + 2$

Shift the Graph **UP** 2

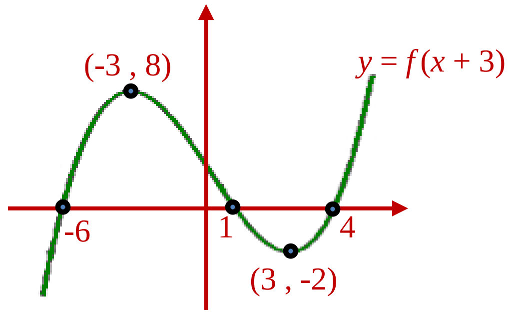


2. Sketch the following graphs:

a) $y = f(x + 3)$

Shift the Graph **LEFT** 3

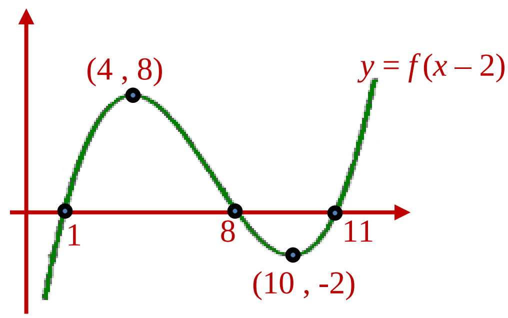
Do the OPPOSITE!!!



b) $y = f(x - 4)$

Shift the Graph **RIGHT** 4

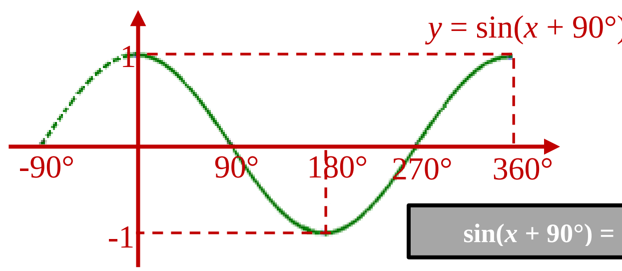
Do the OPPOSITE!!!



c) $y = \sin(x + 90°)$

Shift the Graph **LEFT** 90°

Do the OPPOSITE!!!



$\sin(x + 90°) = \cos x$

Reflections:

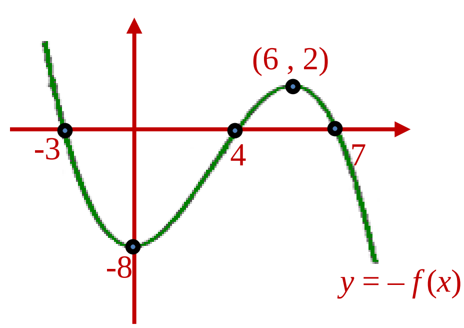
- There are 2 types of **REFLECTIONS** which can be applied to a Graph:.
  - x – axis: $-f(x)$ — Reflects the graph in the x-axis.
    Changes the signs of all the y-coordinates, x stays the same!
  - y – axis: $f(-x)$ — Reflects the graph in the y-axis.
    Changes the signs of all the x-coordinates, y stays the same!

Examples:

3. Sketch the following graphs:

a) $y = -f(x)$

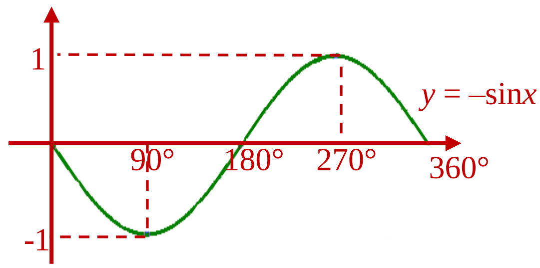
Reflect the Graph in the ***x*-axis**



A point (a , b) will become (a , –b) when reflected in the x-axis. Points on the *x*-axis stay the same.

b) $y = -\sin x$

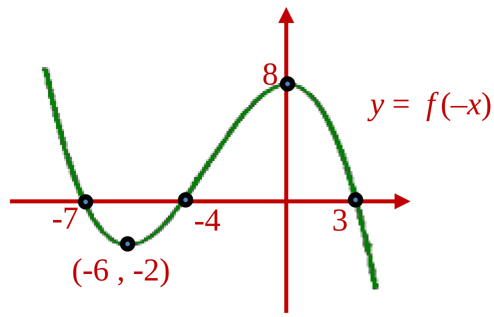
Reflect the Graph in the ***x*-axis**



4. Sketch the following graphs:

a) $y = f(-x)$

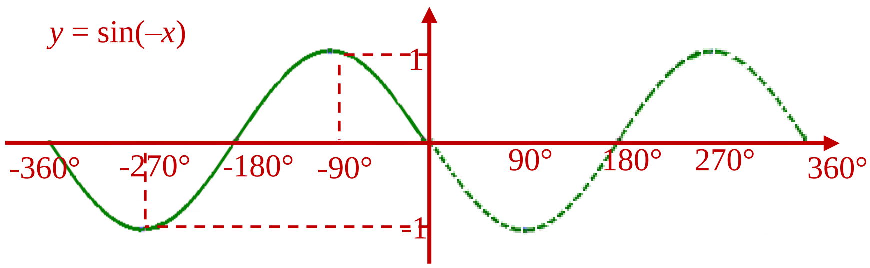
Reflect the Graph in the ***y*-axis**



A point (a , b) will become (–a , b) when reflected in the y-axis. Points on the *y*-axis stay the same.

b) $y = \sin(-x)$

Reflect the Graph in the ***y*-axis**



Notice that the graph of y = sin(–*x*) above is the same as the graph of y = –sin*x* in example 3 above.
This means that we can write sin(–*x*) = –sin*x*. We will see more of this later in the course.

Scaling:

- There are 2 types of **SCALINGS** which can be applied to a Graph:.
  - Horizontally: $f(kx)$ This will cause the **WIDTH** of the graph to be changed as follows:
    STRETCHED if $k < 0$ or **NARROWED** if $k > 0$ ← **Opposite from the way you thimk!!**
    Only the *x*-coordinate will change.
  - Vertically: $k\,f(x)$ This will cause the **HEIGHT** of the graph to be changed as follows:
    STRETCHED if $k > 0$ or **COMPRESSED** if $k < 0$
    Only the *y*-coordinate will change.

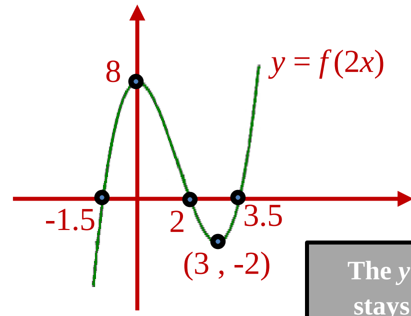
Examples:

5. Sketch the following graphs:

a) $y = f(2x)$

NARROW the Graph horizontally by a factor of 2

Do the OPPOSITE!!!

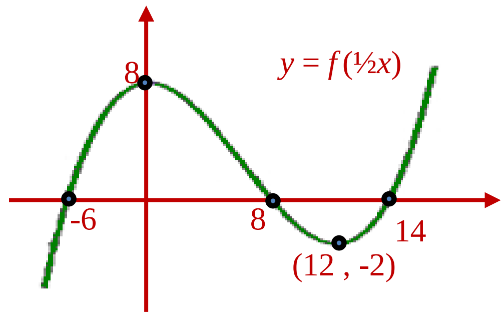


The *y*-coordinate stays the same!!

b) $y = f(\frac{1}{2}x)$

STRETCH the Graph horizontally by a factor of 2

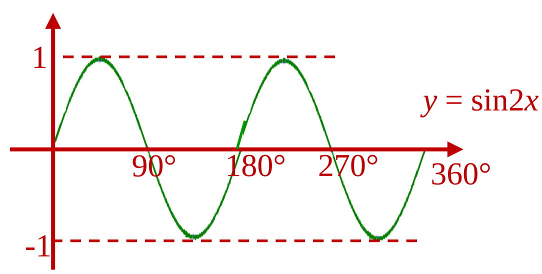
Do the OPPOSITE!!!



c) $y = \sin 2x$

NARROW the Graph horizontally by a factor of 2

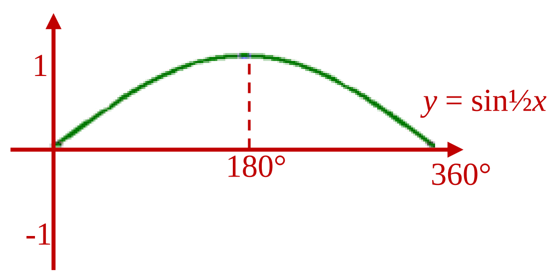
Do the OPPOSITE!!!



d) $y = \sin \frac{1}{2}x$

STRETCH the Graph horizontally by a factor of 2

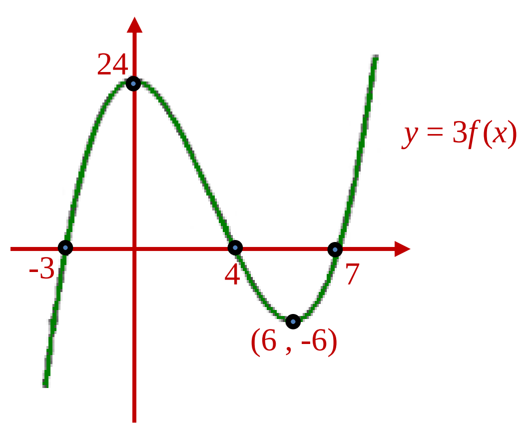
Do the OPPOSITE!!!



6. Sketch the following graphs:

a) $y = 3f(x)$

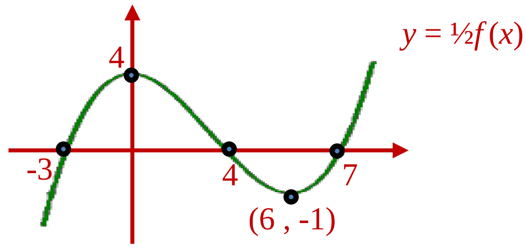
STRETCH the Graph vertically by a factor of 3



The *x*-coordinate stays the same!!

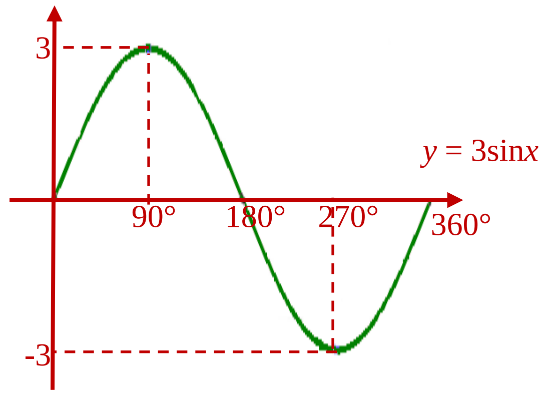
b) $y = ½f(x)$

COMPRESS the Graph
vertically by a factor of 2



c) $y = 3\sin x$

STRETCH the Graph
vertically by a factor of 3



Examples - Combined Questions:

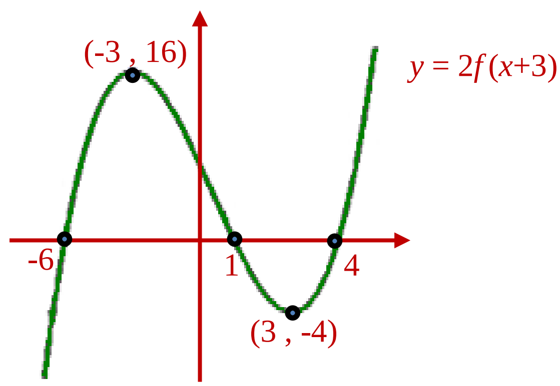
- Usually you are asked to sketch a graph with 2 (or more) transformations as follows:

7. Sketch the following graphs:

a) $y = 2f(x+3)$

STRETCH the Graph
vertically by a factor of 2
then shift the Graph **LEFT** 3

Do the OPPOSITE!!!

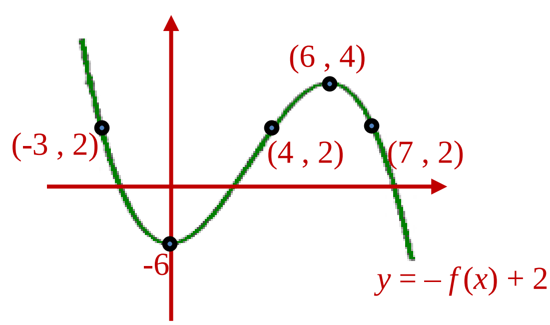


b) $y = 2 - f(x)$

$y = -f(x) + 2$

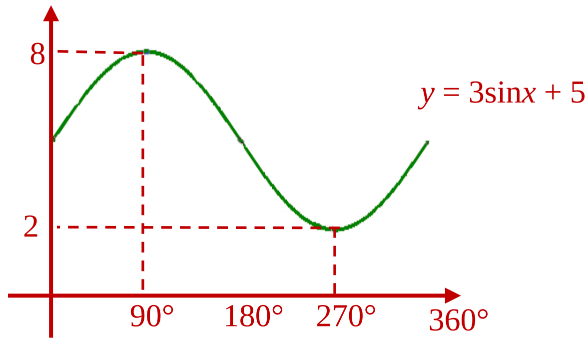
Always rearrange so that $f(x)$ is first

FLIP the Graph vertically
then shift the Graph **UP** 2



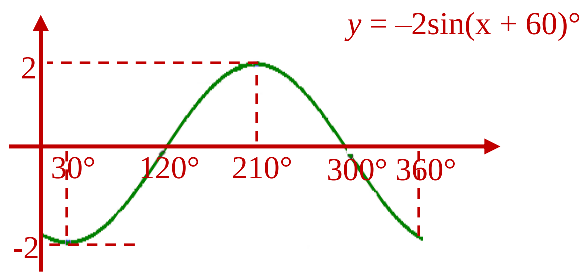
c) y = 3sinx + 5

STRETCH the Graph vertically then shift the Graph **UP** 5



d) y = –2sin(x + 60)°

FLIP & STRETCH the Graph vertically then shift the Graph **LEFT** 60°



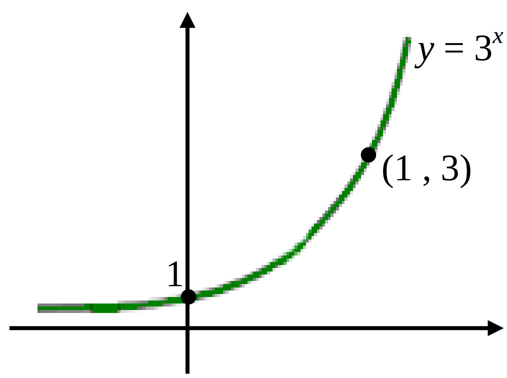
Examples - Related Exponential & Logarithmic Graphs:

- Remember **EXPONENTIAL** graphs, $y = a^x$, always pass through the points: (0 , 1) & (1 , a)

8. The graph of $y = 3^x$, is shown below, sketch the graphs of:

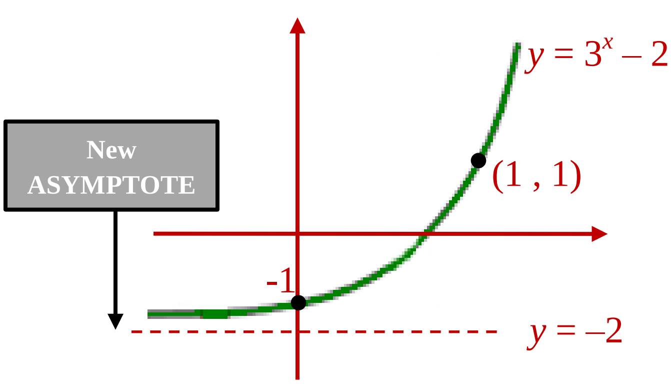
a) $y = 3^x - 2$

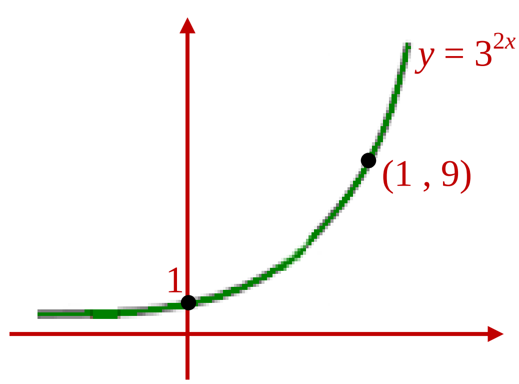
b) $y = 3^{2x}$



a) Shift the Graph **DOWN** 2

(b) $y = 3^{2x} = (3^2)^x = 9^x$
Graph passes through (0 , 1) & (1 , 9)



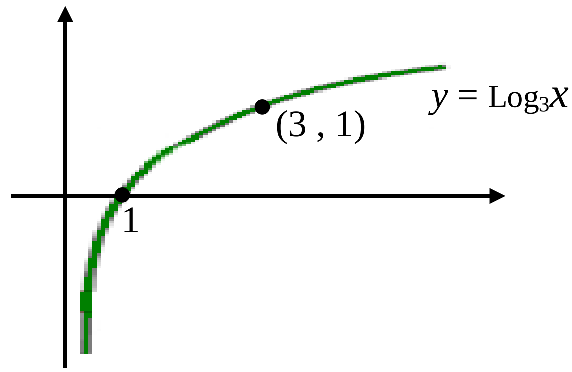


- Remember **<u>LOGARITHMIC</u>** graphs, $y = \text{Log}_a x$, always pass through the points: (1 , 0) & (a , 1)

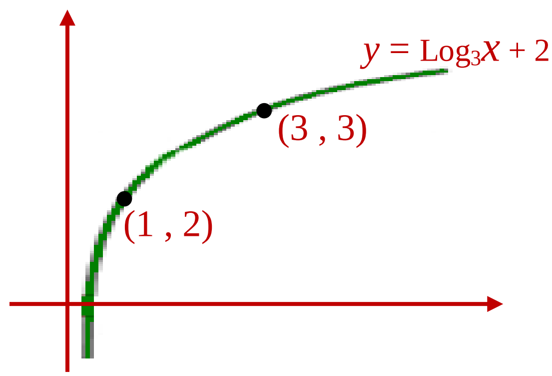
9. The graph of $y = \text{Log}_3 x$, is shown below, sketch the graphs of:

a) $y = \text{Log}_3 x + 2$

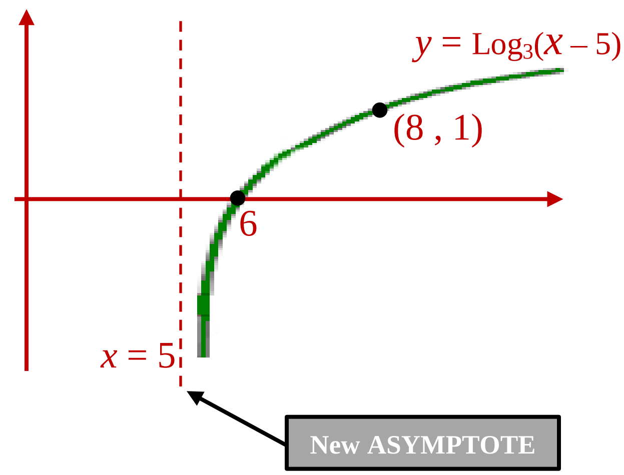
b) $y = \text{Log}_3(x - 5)$



a) Shift the Graph **<u>UP</u>** 2



(b) Shift the Graph **<u>RIGHT</u>** 5



<u>Examples - Finding Related Graph Equations:</u>

- You may also be given the Graph and asked to find the equation of the curve.

10. For the graph opposite find the values of a and b and then state the curves equation.

Use (0 , 4): $4 = a^0 + b \Longrightarrow 4 = 1 + b \Longrightarrow b = 3$

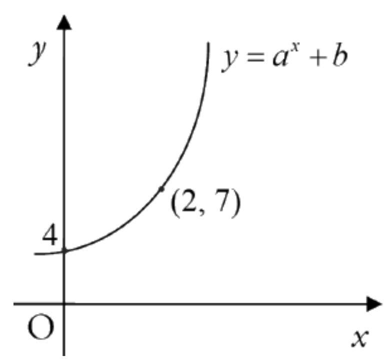
Use (2 , 7): $7 = a^2 + 3 \Longrightarrow 4 = a^2 \Rightarrow a = \sqrt{4}$

$\Rightarrow a = \pm 2$

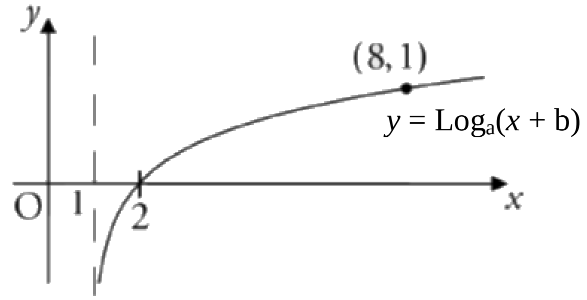
$\Rightarrow a = 2$

Since a > 0

So $y = 2^x + 3$



11. For the graph opposite find the values of a and b and then state the curves equation.



Log curve usually passes through (1 , 0) but this passes through (0 , 2) so it has been shifted right by 1 $\Longrightarrow b = -1$

Use (8 , 1): $y = \text{Log}_a(x - 1)$

$\Longrightarrow 1 = \text{Log}_a(8 - 1)$

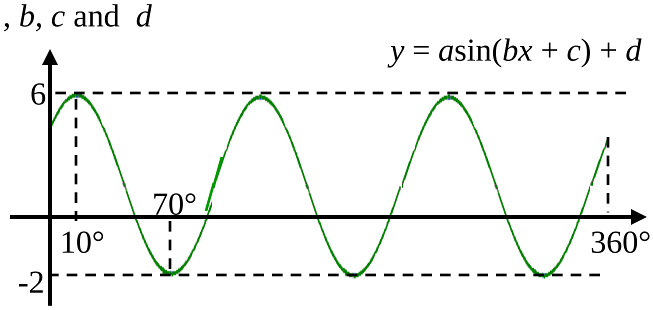
$\Longrightarrow 1 = \text{Log}_a 7$

$\Longrightarrow a = 7$

So $y = \text{Log}_7(x - 1)$

Since $\text{Log}_a a = 1$

12. For the graph opposite find the values of a , b, c and d and then state the curves equation.



a is half the height of the graph, so: $a = 4$

b is the number of cycles between 0 & 360, so: $b = 3$

d is how much it has been shifted vertically, so: $d = 2$

c is how much it has been shifted horizontally: Sin graph is at its maximum at 90°
So sin3x is at its maximum at 30°
This curve is at its maximum at 10°
So it has been shifted 20° to the left, so $c = 20°$

So $y = 4\sin(3x + 20°) + 2$