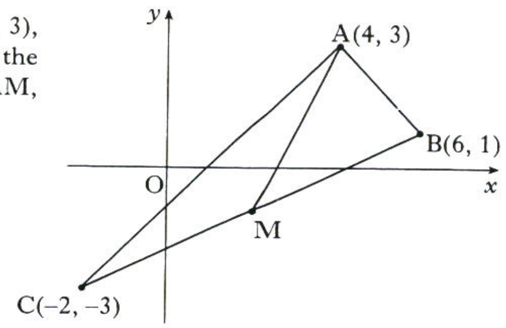
SCOTTISH CERTIFICATE OF EDUCATION 1998

THURSDAY, 7 MAY
9.30 AM – 11.30 AM

MATHEMATICS
HIGHER GRADE
Paper I

All questions should be attempted *Marks*

1. A triangle ABC has vertices A(4, 3), B(6, 1) and C(–2, –3) as shown in the diagram. Find the equation of AM, the median from A. **(3)**



2. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **(4)**

3. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by

$$\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k},\quad \boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k} \text{ and } \boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}.$$

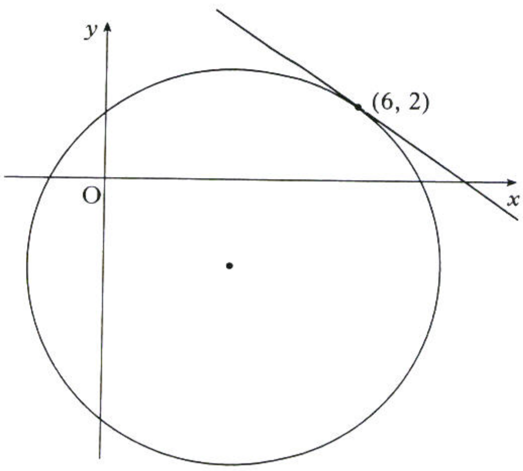
(*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. **(2)**

(*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ **(1)**

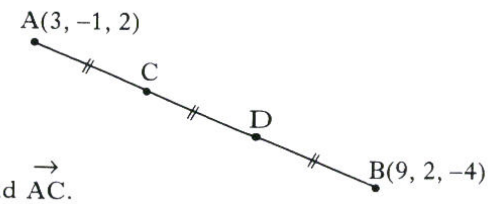
(*c*) Find $|\boldsymbol{r}|$. **(1)**

4. The circle shown has equation $(x-3)^2 + (y+2)^2 = 25$.

Find the equation of the tangent at the point (6, 2). **(4)**



5. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).



(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Find the coordinates of C and D. **(2)**

Marks

6. The functions f and g are defined on a suitable domain by

$$f(x) = x^2 - 1 \text{ and } g(x) = x^2 + 2.$$

(*a*) Find an expression for $f(g(x))$. **(2)**

(*b*) Factorise $f(g(x))$. **(2)**

7. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.
Find the exact value of

(*a*) $\sin 2A$ **(2)**

(*b*) $\cos 2A$ **(1)**

(*c*) $\sin(2A + B)$. **(2)**

8. Two sequences are defined by these recurrence relations

$$u_{n+1} = 3u_n - 0 \cdot 4 \text{ with } u_0 = 1, \qquad v_{n+1} = 0 \cdot 3v_n + 4 \text{ with } v_0 = 1.$$

(*a*) Explain why only one of these sequences approaches a limit as $n \to \infty$. **(1)**

(*b*) Find algebraically the exact value of the limit. **(2)**

(*c*) For the other sequence, find

(i) the smallest value of n for which the n^{th} term exceeds 1000, and

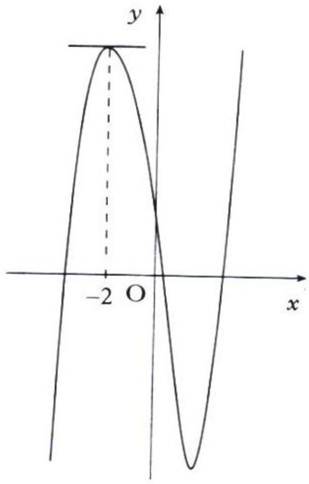
(ii) the value of that term. **(2)**

9. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1, \quad 0 \le x < 2\pi$. **(4)**

10. A curve, for which $\frac{dy}{dx} = 6x^2 - 2x$, passes through the point $(-1, 2)$.
Express y in terms of x. **(3)**

11. The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis.

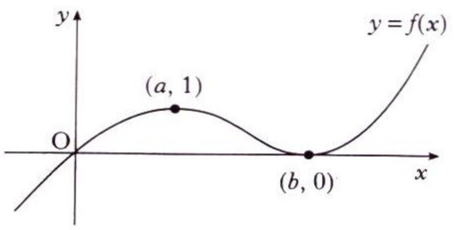
Find the value of k. **(4)**



12. Evaluate $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$. **(5)**

Marks

13. A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$.



(a) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. **(3)**

(b) On a separate diagram sketch the graph of $y = f'(x)$. **(2)**

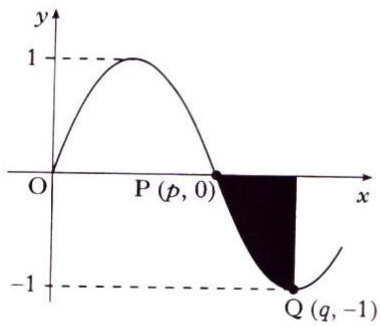
(c) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$.

Use this information to write down the coordinates of a point on the graph of $y = f'(x)$. **(1)**

14. Differentiate $2\sqrt{x}(x+2)$ with respect to x. **(4)**

15. A sketch of part of the graph of $y = \sin 2x$ is shown in the diagram.

The points P and Q have coordinates $(p, 0)$ and $(q, -1)$.



(a) Write down the values of p and q. **(1)**

(b) Find the area of the shaded region. **(4)**

16. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **(3)**

17. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

(a) Find the speed of the ball after 1 second. **(3)**

(b) For how many seconds is the ball travelling upwards? **(2)**

18. (a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **(3)**

(b) Show that there are no real roots for θ. **(1)**

19. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **(4)**

[END OF QUESTION PAPER]