2009 Paper 1 Solutions

Objective Questions

Qu.	Key	Item no.	solution
1.01	A	999	• $u_2 = 3\times 2+4=10$ • $\therefore u_5 = 3\times 10+4=34$
1.02	B	153	$x^2+y^2+8x+6y-75=0$ • $r=\sqrt{(-4)^2+(-3)^2-(-75)}$ • $r=10$
1.03	D	950	• $S=\left(\frac{-1+3}{2},\frac{4+6}{2}\right)=(1,5)$ • $m_{PQ}=\frac{5--2}{1--3}=\frac{7}{4}$
1.04	C	60	• $\frac{dy}{dx}=15x^2-12$ • at $x=1$, gradient $=15-12=3$
1.05	B	1201	• $ST=\sqrt{(2-5)^2+(3--1)^2}$ $ST=5$ • $m_{ST}=\frac{3--1}{2-5}=-\frac{4}{3}$
1.06	A	1239	• $L=0.7L+10$ • $L=\frac{10}{0.3}=\frac{100}{3}$
1.07	A	63	• $\cos(2x)=2\cos^2(x)-1$ • $2\times\left(\frac{1}{\sqrt{5}}\right)^2-1=-\frac{3}{5}$
1.08	D	1081	• $f(x)=\frac{1}{4}x^{-3}$ • $f'(x)=-\frac{3}{4}x^{-4}$
1.09	A	1901	• $x^2+(2x)^2=5$ • $5x^2=5,\ \ x=\pm 1$
1.10	B	1903	• $x=3,\ y=\log(3-2)=0$ *so B* • $x=7,\ y=\log_5(7-2)=1$

Qu.	Key	Item no.	solution
1.11	B	1145	• $\sin x=\frac{\sqrt{5}}{4}$: 2 *solutions* • $\sin x=-1$: 1 *solution*
1.12	C	1313	• $b^2-4ac=73>0$ • *roots are real and distinct*
1.13	B	1146	• $\tan a^\circ=\frac{1}{\sqrt{3}}$ *so* $a=30$ • $k^2=1+3$ *so* $k=2$
1.14	C	1172	• $f_{max}=2\times 1+5=7$ • $f_{min}=2\times(-1)+5=3$
1.15	A	1396	• angle at x-axis $=\frac{\pi}{3}$ • $m_{GH}=\tan\frac{\pi}{3}=\sqrt{3}$
1.16	B	1148	• integrate : x^4-3x^3 • limits : $-[\ldots]_0^1$
1.17	A	1133	• $\lvert u\rvert=\sqrt{(-3)^2+4^2}=5$ • *a unit vector* : $\frac{1}{5}(-3i+4j)$
1.18	D	394	• $-\frac{1}{2}\left(4-3x^2\right)^{-\frac{3}{2}}$ • multiplied by $-6x$
1.19	C	1002	• $(2+x)(3-x)<0$ *solution is either* $-2<x<3$ ***or*** $x<-2,\ x>3$ • $x=0$ *is FALSE so* $x<-2$ *and* $x>3$
1.20	C	161	• $\frac{dA}{dr}=4\pi r+6\pi$ • $\frac{dA}{dr}_{r=2}=8\pi+6\pi$ $=14\pi$

Qu. 21

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

•1	ic	interpret x-intercept
•2	pd	find gradient (of QR)
•3	ss	know and use $m_1 m_2 = -1$
•4	ic	state equ. of altitude
•5	ic	state equ. of line (QR)
•6	ss	prepare to solve sim. equ.
•7	pd	solve for x
•8	pd	solve for y

Primary Method : Give 1 mark for each •

•1	$P = (-3, 0)$	see Notes 1, 2
•2	$m_{QR} = -2$ or equivalent	
•3	$m_{alt} = \frac{1}{2}$ s / i by •4	
•4	$alt : y - 0 = \frac{1}{2}(x + 3)$	see Note 4
•5	$QR : y + 2 = -2(x - 8)$ *or* $y - 6 = -2(x - 4)$	
•6	*e.g.* $x - 2y = -3$ *and* $2x + y = 14$	see Note 5 & Options
•7	$x = 5$	
•8	$y = 4$	

Qu. 22

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

In this question expressing vectors as coordinates and vice versa is treated as bad form - do not penalise.

•1	ss	use vector approach
•2	ic	compare two vectors
•3	ic	complete proof
•4	ic	state ratio
•5	ss	use vector approach
•6	ss	know scalar product = 0 for ⊥ vectors
•7	pd	start to solve
•8	pd	complete

Primary Method : Give 1 mark for each •

•1	$\overrightarrow{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ *or* $\overrightarrow{EF} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$	see Note 1
•2	2nd column vector ***and*** $\overrightarrow{DE} = 3\overrightarrow{EF}$ *(or equiv.)*	
•3	$\overrightarrow{DE}$ and $\overrightarrow{EF}$ have common point and common direction hence D, E and F collinear	see Note 2
•4	3 : 1 stated explicitly	
•5	$\overrightarrow{GE} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$	
•6	$\overrightarrow{DE}.\overrightarrow{GE} = 0$	s / i by •7
•7	$-9(1-k) + 6 \times (-3) + 12 \times (-3)$	
•8	$k = 7$	

Qu. 23

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

•1	ic	scaling parallel to x-axis
•2	ic	annotate graph
•3	ss	correct order for refl(x) & trans
•4	ic	start to annotate final sketch
•5	ic	complete annotation

Primary Method : Give 1 mark for each •

3 points : the origin, (1, 8) and (–2, 8)

•1	sketch and 1 point correct
•2	other two points correct
•3	reflect in x-axis, then vertical trans. s / i by •4

final points : (0, 1), (1, – 7) and (–2, – 7)

•4	sketch and 1 final point correct
•5	the other two final points correct

Qu. 24

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

•1	ss	expand compound angle
•2	ic	substitute exact values
•3	pd	process to a single fraction
•4	ic	start proof
•5	ic	complete proof
•6	ss	identify steps
•7	ic	start process (identify 'A' & 'B')
•8	ic	substitute
•9	pd	process

Primary Method : Give 1 mark for each •

•1 $\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ **s / i by** •2

•2 $\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}$

•3 $\frac{\sqrt{3}+1}{2\sqrt{2}}$ *or equivalent*

•4 $\sin A\cos B+\cos A\sin B+\ldots$

•5 $\ldots\ldots+\sin A\cos B-\cos A\sin B$ and complete

•6 $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$ **stated explicitly**

and *A is* $\frac{\pi}{3}$, *B is* $\frac{\pi}{4}$ **s / i by** •7

•7 $2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$

•8 $2\times\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$

•9 $\frac{\sqrt{6}}{2}$ $\left(\text{accept } \sqrt{\frac{3}{2}} \text{ or } \frac{\sqrt{3}}{\sqrt{2}} \text{ but not } \frac{2\sqrt{3}}{2\sqrt{2}}\right)$