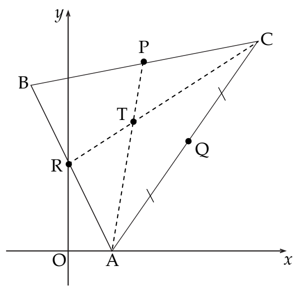
Question	Answer
1	A
2	C
3	D
4	A
5	B
6	D
7	C
8	B
9	C
10	B
11	D
12	A
13	B
14	C
15	C
16	A
17	B
18	B
19	C
20	A

Summary		
	A	5
	B	6
	C	6
	D	3

21 Triangle ABC has vertices A (4, 0), B (−4, 16) and C (18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T (6, 12).



(*a*)	Find the equation of median BQ.	3
(*b*)	Verify that T lies on BQ.	1

Generic Scheme			Illustrative Scheme
21 (a)			
•1	ss	know and find midpoint of AC	•1 (11, 10)
•2	pd	calculate gradient of BQ	•2 $-\frac{6}{15}$ or equivalent
•3	ic	state equation	•3 $y-16=-\frac{2}{5}(x-(-4))$ **or** $y-10=-\frac{2}{5}(x-11)$

Notes

1. Candidates who do not use a midpoint lose •2 and •3.
2. There is no need to simplify m_{BQ} for •2. It must, however, be simplified before •3 can be awarded.
   Do not award •3 for $6x+15y-216=0$, although •3 would be awarded for $6x+15y-216=0$ then $2x+5y-72=0$.
3. If m_{BQ} cannot be simplified, due to an error, then •3 is still available.
4. •3 is available for using $y=mx+c$ where $m=-\frac{2}{5}$ **and** $c=\frac{72}{5}$.
5. Accept $y=-0\cdot4x+14\cdot4$.
6. Candidates who find the equations of AP or CR can only gain 1 mark.

   AP : $y-0=6(x-4)$ or $y-12=6(x-6)$ CR : $y-20=\frac{2}{3}(x-18)$ or $y-12=\frac{2}{3}(x-6)$

Generic Scheme			Illustrative Scheme
21 (b)			
•4	ic	substitute in for T and complete	•4 e.g. Substitution : $2(6)+5(12)=12+60=72$ Gradients : $m_{BT}=-\frac{4}{10}=-\frac{2}{5}=m_{BQ}$ Vectors : $\overrightarrow{BT}=\begin{pmatrix}10\\-4\end{pmatrix}$, $\overrightarrow{TQ}=\begin{pmatrix}5\\-2\end{pmatrix}$ and $\overrightarrow{BT}=2\overrightarrow{TQ}$

Notes

7. •4 is available as follow through with an appropriate communication statement, e.g. 'T does not lie on BQ'.
8. Statements such as 'PA, RC and BQ are all medians and therefore all share the same point T' do not gain •4.
9. Since only 1 mark is available here, do not penalise the omission of any reference to a "common point" or "parallel".

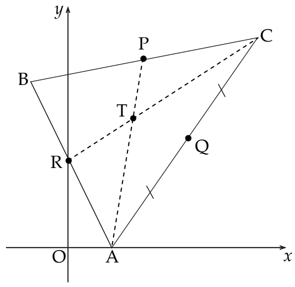
Regularly occurring responses

Gradient approach : (b) $m_{BT}=-\frac{4}{10}=-\frac{2}{5}=m_{BQ}$ leading to 2 : 1 in (c), without further working, gains •4 and •6 but loses •5.

but (b) $m_{BQ}=-\frac{6}{15}$ and $m_{TQ}=-\frac{2}{5}$ leading to $m_{BQ}=3m_{TQ}$ so T lies on BQ leading to 2 : 1 in (c), without further working, loses •4 and •5 but gains •6.

21 Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



(*c*) Find the ratio in which T divides BQ. **2**

Generic Scheme	Illustrative Scheme

21 (c)

$\bullet^5$ ss valid method for finding the ratio

$\bullet^6$ ic complete to simplified ratio

For 2 : 1 **without** working, only $\bullet^6$ is awarded.
Be aware that the working may appear in (b).
Some candidates obtain 2 : 1 from erroneous working thus losing $\bullet^6$.

Method 1 : Vector approach

$\bullet^5$ e.g. $\overrightarrow{BT} = \begin{pmatrix}10\\-4\end{pmatrix}$ and $\overrightarrow{TQ} = \begin{pmatrix}5\\-2\end{pmatrix}$

$\bullet^6$ 2 : 1

Method 2 : “Stepping out” approach

$\bullet^5$ e.g. B —10— T —5— Q or

B	T	Q
– 4	6	11
10	5	

$\bullet^6$ 2 : 1

Method 3 : Distance Formula approach

$\bullet^5$ e.g. $d_{BT} = \sqrt{116}$ and $d_{TQ} = \sqrt{29}$

$\bullet^6$ 2 : 1

Notes

10. Any evidence of appropriate steps, e.g. 10 and 5 or 4 and 2 but not 2 and 1, can be awarded $\bullet^5$ leading to $\bullet^6$,
e.g. B —2— T —1— Q is not sufficient on its own and so loses $\bullet^5$ but gains $\bullet^6$.

11. $\sqrt{116} : \sqrt{29}$ with no further simplification may be awarded $\bullet^5$ but not $\bullet^6$.

12. In this question working for (c) may appear in (b), where the working appears for $\bullet^4$.

Regularly occurring responses

Response 1

(b) $\overrightarrow{BT} = \begin{pmatrix}10\\-4\end{pmatrix}$ $\overrightarrow{TQ} = \begin{pmatrix}5\\-2\end{pmatrix}$ ✓ $\bullet^5$

$\overrightarrow{BT} = 2\overrightarrow{TQ}$ ✓ $\bullet^4$

(c) 2 : 1 ✓ $\bullet^6$

3 marks out of 3

Response 2

(b) $\overrightarrow{QT} = \begin{pmatrix}-5\\2\end{pmatrix}$ $\overrightarrow{BQ} = \begin{pmatrix}15\\-6\end{pmatrix}$ ✓ $\bullet^5$ $= -3\overrightarrow{QT}$ ✓ $\bullet^4$

(c) 2 : 1 ✓ $\bullet^6$

3 marks out of 3

Response 3

(b) $\overrightarrow{QT} = \begin{pmatrix}-5\\2\end{pmatrix}$ $\overrightarrow{TB} = \begin{pmatrix}-10\\4\end{pmatrix}$ ✓ $\bullet^5$

✓ $\bullet^4$

(c) $\begin{pmatrix}-10\\4\end{pmatrix} = 2\begin{pmatrix}-5\\2\end{pmatrix}$ so 2 : 1 ✓ $\bullet^6$

3 marks out of 3

Response 4

(b) $\overrightarrow{BT} = \begin{pmatrix}10\\-4\end{pmatrix}$ $\overrightarrow{TQ} = \begin{pmatrix}5\\-2\end{pmatrix}$ ✓ $\bullet^5$ so $2\overrightarrow{BT} = \overrightarrow{TQ}$ ✗ $\bullet^4$ (c) 2 : 1 ✗ $\bullet^6$ but 1 : 2 would have gained $\bullet^6$

22	(*a*) (i) Show that $(x-1)$ is a factor of $f(x)=2x^3+x^2-8x+5$. (ii) Hence factorise $f(x)$ fully.	**5**
	(*b*) Solve $2x^3+x^2-8x+5=0$.	**1**

Generic Scheme			Illustrative Scheme	
22 (a)			**Method 1** : Using synthetic division	
•1	ss	know to use $x=1$	•1 $\begin{array}{c\|cccc} 1 & 2 & 1 & -8 & 5 \\ \hline \end{array}$	
•2	ic	complete evaluation	•2 $\begin{array}{c\|cccc} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & 0 \end{array}$	
•3	ic	state conclusion	•3 $(x-1)$ is a factor	*see note 2*
•4	pd	find quadratic factor	•4 $2x^2+3x-5$	*stated, or implied by* •5
•5	pd	factorise completely	•5 $(x-1)(x-1)(2x+5)$	*stated explicitly*
			Method 2 : Using substitution and inspection	
			•1 know to use $x=1$	
			•2 $2+1-8+5=0$	
			•3 $(x-1)$ is a factor	*see note 2*
			•4 $(x-1)(2x^2+3x-5)$	
			•5 $(x-1)(x-1)(2x+5)$	*stated explicitly*

Notes

1. Communication at •3 must be consistent with working at •2.
   i.e. candidate's working must arrive legitimately at zero before •3 is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-1)$ is not a factor'.
2. For •3, minimum acceptable statement is 'factor'.
   Unacceptable statements : $x=1$ is a factor, $(x+1)$ is a factor, $x=1$ is a root, $(x-1)$ is a root etc.
3. At •5 the expression may be written as $(x-1)^2(2x+5)$.

22 (b)				These may appear in the working at (a).
•6	ic	state solutions	•6 $x=1$ **and** $x=-\frac{5}{2}$ or $-2\cdot 5$ or $-2\frac{1}{2}$	

Notes

4. From (a) $(x-1)(x-1)(2x+5)$ leading to $x=1, x=-\frac{5}{2}$ then $(1, 0)$ and $\left(-\frac{5}{2}, 0\right)$ gains •6.

   However, $(x-1)(x-1)(2x+5)$ leading to $(1, 0)$ and $\left(-\frac{5}{2}, 0\right)$ **only** does not gain •6.

5. From (a) $(x-1)(2x+5)$ only leading to $x=1, x=-\frac{5}{2}$ does not gain •6 as equation solved is not a cubic, but $(x-1)(x+1)(2x-5)$ leading to $x=1, x=-1$ and $x=\frac{5}{2}$ gains •6 as follow through from a cubic equation.

22 (*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.
Write down the coordinates of H. **1**

Generic Scheme			Illustrative Scheme	
22 (c)				
Method 1 : Equating curve and line			**Method 1** : Equating curve and line	
•7	ss	set $y_{\text{CURVE}} = y_{\text{LINE}}$	•7 $2x^3 + x^2 - 6x + 2 = 2x - 3$	*stated explicitly*
•8	ic	express in standard form	•8 $2x^3 + x^2 - 8x + 5$ $= 0$	*see note 6*
•9	ss	compare with (a) or factorise	•9 $(x-1)(x-1)(2x+5)$ $= 0$	
•10	ic	identify x_G	•10 $x = 1$	
•11	pd	evaluate y_G	•11 $y = -1$	
Method 2 : Differentiation			**Method 2** : Differentiation	
•7	ss	know to and differentiate curve	•7 $6x^2 + 2x - 6$	
•8	ic	set derivative to gradient of line	•8 $6x^2 + 2x - 6 = 2$	
•9	pd	solve quadratic equation	•9 $x = -\frac{4}{3}$ and 1	
•10	ss	process to identify x_G	•10 at $x = 1$ evaluate y_{CURVE} **and** y_{LINE}	
•11	ic	complete to $y_{\text{CURVE}} = y_{\text{LINE}}$	•11 $y = -1$ from both curve and line	

Notes

In method 1:

6. •8 is only available if '= 0' appears at either the •8 or •9 stage.
7. •9 , •10 and •11 are only available via the working from •7 and •8 .
8. If $(x-1)(x-1)(2x+5)$ does not appear at •9 stage, it can be implied by •5 **and** •10 .
9. At •9 a quadratic used from (a) may gain •9 , •11 and •12 but a quadratic from •8 may gain •11 and •12 only.
10. If G and H are interchanged then •10 is lost but •11 and •12 are still available.
11. Candidates who obtain three distinct factors at •9 can gain •11 for evaluating **all** y values, but lose •10 and •12.
12. A repeated factor at •5 **or** •9 stage is required for •10 to be awarded without justification.

In both methods:

13. All marks in (c) are available as a result of differentiating $2x^3 + x^2 - 6x + 2$ and solving this equal to 2 (from method 2).
    Only marks •7 and •8 (from method 1) are available to those candidates who choose to differentiate $2x^3 + x^2 - 8x + 5$ and solve this equal to 0.
14. Candidates may choose a combination of making equations equal and differentiation.

22 (d)				
•12	pd	state solution	•12 $\left(-\frac{5}{2}, -8\right)$	*may appear in (c)*

Notes

15. Method 2 from (c) would not yield a value for H and so •12 is not available.

23	(*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x-2y=0$. (i) Show that $\tan a=\frac{3}{2}$. (ii) Find the value of $\sin a$.	**4**
	(*b*) A second right angled triangle is added as shown in Diagram 2. The line OB has equation $3x-4y=0$. Find the values of $\sin b$ and $\cos b$.	**4**

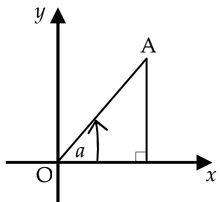


Diagram 1

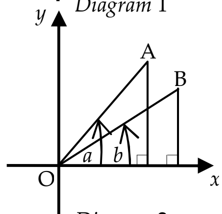


Diagram 2

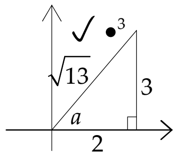
Generic Scheme			**Illustrative Scheme**	
23 (a)				
•1	ss	write in slope/intercept form	•1 $y=\frac{3}{2}x$ or $y=1\cdot5x$	*stated explicitly*
•2	ic	connect gradient and $\tan a$	•2 $m=\frac{3}{2}$ and $\tan a=\frac{3}{2}$ **or** $m=\tan a$ and $\tan a=\frac{3}{2}$	
•3	pd	calculate hypotenuse	•3 $\sqrt{13}$	*stated, or implied by* •4
•4	ic	state value of sine ratio	•4 $\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$	*may not appear until* (c)

Notes

1. •4 is only available if $-1\le\sin a\le1$.
2. Only numerical answers are acceptable for •3 and •4.

Regularly occurring responses

Response 1



$3x-2y=0$

$3\times(2)-2\times(3)=0$ ∧ •1

$\tan a=\frac{3}{2}$ ✗ •2

$\sin a=\frac{3}{\sqrt{13}}$ ✓ •4

2 marks out of 4

23 (b)				
•5	ss	determine $\tan b$	•5 $\tan b=\frac{3}{4}$	*stated, or implied by* •6
•6	ss	know to complete triangle	•6 right angled triangle with 3 and 4 correctly shown	
•7	pd	determine hypotenuse	•7 5	*stated, or implied by* •8
•8	ic	state values of sine and cosine ratios	•8 $\sin b=\frac{3}{5}$ and $\cos b=\frac{4}{5}$	*may not appear until* (c)

Notes

3. •8 is only available if $-1\le\sin b\le1$ **and** $-1\le\cos b\le1$.
4. $\sin b=\frac{3}{5}$ and $\cos b=\frac{4}{5}$ without working is awarded 3 marks only.
5. Only numerical answers are acceptable for •7 and •8.

23	(c) (i) Find the value of $\sin(a-b)$. (ii) State the value of $\sin(b-a)$.	**4**

Generic Scheme | **Illustrative Scheme**

23 (c)

•[9]	ss	know to use addition formula	•[9]	$\sin a\cos b-\cos a\sin b$
•[10]	ic	substitute into expansion	•[10]	$\frac{3}{\sqrt{13}}\times\frac{4}{5}-\frac{2}{\sqrt{13}}\times\frac{3}{5}$
•[11]	pd	evaluate sine of compound angle	•[11]	$\frac{6}{5\sqrt{13}}$
•[12]	ss	use $\sin(-x)=-\sin x$	•[12]	$-\frac{6}{5\sqrt{13}}$

Notes

6. $\sin(A-B)=\sin A\cos B-\cos A\sin B$, or just $\sin A\cos B-\cos A\sin B$, with no further working does not gain •[9].
7. Candidates should not be penalised further at •[10], •[11] and •[12] for values of sine and cosine outside the range -1 to 1.
8. Candidates who use $\sin(a-b)=\sin a-\sin b$ lose •[9], •[10] and •[11] but can gain •[12], as follow through, only for a non-zero answer which is obtained from the result $\sin(-x)=-\sin x$.
9. Treat $\sin\frac{3}{\sqrt{13}}\cos\frac{4}{5}-\cos\frac{2}{\sqrt{3}}\sin\frac{3}{5}$ as bad form only if 'sin' and 'cos' subsequently disappear.
10. It is acceptable to work through the whole expansion again for •[12].

Regularly occurring responses

Response 1

$\sin(a-b)=\sin a-\sin b$ ✗ •[9]
$=6-6$ ✗ •[10]
$=0$ ✗ •[11]

$\sin(b-a)=0$ ✗ •[12]

0 marks out of 4

Response 2

$\sin a=3 \quad \cos a=2$
$\sin b=3 \quad \cos b=4$

Marks lost in (a) or (b)

$\sin(a-b)=\sin a\cos b-\cos a\sin b$ ✓ •[9]
$=3\times4-2\times3$ ✓ •[10]
$=6$ ✓ •[11]

Eased - not dealing with fraction containing a surd.

$\sin(b-a)=-6$ ✓ •[12]

3 marks out of 4

Response 3

From (a) and (b) $\sin a=\frac{2}{3} \quad \cos a=\frac{1}{3}$
$\sin b=\frac{2}{3} \quad \cos b=\frac{3}{5}$

(c) (i) $\sin(a-b)=\sin a\cos b-\cos a\sin b$ ✓ •[9]
$=\frac{2}{3}\times\frac{3}{5}-\frac{1}{3}\times\frac{2}{5}$ ✓ •[10]
$=\frac{4}{15}$ ✓ •[11]

(ii) $\sin(b-a)=\sin b\cos a-\cos b\sin a$
$=\frac{2}{5}\times\frac{1}{3}-\frac{3}{5}\times\frac{2}{3}$
$=-\frac{4}{15}$ ✓ •[12]

3 marks out of 4

Here the working was not necessary; the answer would gain •[12], provided it is non zero.

Response 4

(i) $\sin(a-b)=\sin a\sin b-\cos a\cos b$ ✗ •[9]
$=\frac{3}{\sqrt{13}}\times\frac{3}{5}-\frac{2}{\sqrt{13}}\times\frac{4}{5}$ ✓ •[10]
$=\frac{1}{5\sqrt{13}}$ ✓ •[11]

(ii) $\sin(b-a)=-\frac{1}{5\sqrt{13}}$ ✓ •[12]

3 marks out of 4