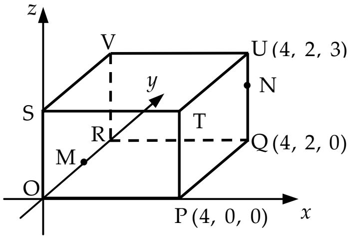
1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$,

Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $\text{UN} = \frac{1}{3}\text{UQ}$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express the vectors $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

Treat as bad form, coordinates written as components and vice versa, throughout this question.

Generic Scheme			Illustrative Scheme
1 (a)			
$\bullet^1$	ic	interpret midpoint for M	$\bullet^1$ (0, 1, 0)
$\bullet^2$	ic	interpret ratio for N	$\bullet^2$ (4, 2, 2)

1 (b)			
$\bullet^3$	ic	intepret diagram	$\bullet^3$ $\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$
$\bullet^4$	pd	process vectors	$\bullet^4$ $\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

Using evidence from (a) or may have been taken directly from diagram.

Notes

1. V is the point $(0, 2, 3)$, which may or may not appear in the working to (b).

Regularly occurring responses

Response 1

(a) M(2, 0, 0) ✗ $\bullet^1$ N(4, 2, −1) ✗ $\bullet^2$

0 marks out of 2

(b) $\overrightarrow{VM} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$ $\cancel{\checkmark}$ $\bullet^3$

Consistent with (a)

$\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ ✓ $\bullet^4$

From diagram

2 marks out of 2

Response 2

Incorrect V stated

(b) V(0, 3, 2)

$\overrightarrow{VM} = \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}$ ✗ $\bullet^3$

$\overrightarrow{VN} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$ $\cancel{\checkmark}$ $\bullet^4$

1 mark out of 2

Response 3

(a) M(0, 2, 0) ✗ $\bullet^1$ N(4, 2, 2) ✓ $\bullet^2$

1 mark out of 2

(b) $\overrightarrow{VM} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix}$ ✗ $\bullet^3$

$\overrightarrow{VN} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ ✗ $\bullet^4$

V (4, 2, 3) used in both but not stated

0 marks out of 2

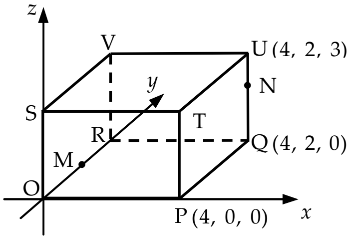
1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$,

Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $\text{UN} = \frac{1}{3}\text{UQ}$.



(*c*) Calculate the size of angle MVN. **5**

Treat as bad form, coordinates written as components and vice versa, throughout this question.

Generic Scheme	**Illustrative Scheme**
1 (c)	
Method 1 : Vector Approach	**Method 1** : Vector Approach
$\bullet^5$ ss know to use scalar product	$\bullet^5$ $\cos M\hat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$ *stated, or implied by* $\bullet^9$
$\bullet^6$ pd find scalar product	$\bullet^6$ $\overrightarrow{VM}.\overrightarrow{VN} = 3$
$\bullet^7$ pd find magnitude of a vector	$\bullet^7$ $\lvert\overrightarrow{VM}\rvert = \sqrt{10}$
$\bullet^8$ pd find magnitude of a vector	$\bullet^8$ $\lvert\overrightarrow{VN}\rvert = \sqrt{17}$
$\bullet^9$ pd evaluate angle	$\bullet^9$ $76\cdot7^\circ$ or $1\cdot339$ rads or $85\cdot2$ grads
Method 2 : Cosine Rule Approach	**Method 2** : Cosine Rule Approach
$\bullet^5$ ss know to use cosine rule	$\bullet^5$ $\cos M\hat{V}N = \dfrac{VM^2 + VN^2 - MN^2}{2\times VM\times VN}$ *stated, or implied by* $\bullet^9$
$\bullet^6$ pd find magnitude of a side	$\bullet^6$ $VM = \sqrt{10}$
$\bullet^7$ pd find magnitude of a side	$\bullet^7$ $VN = \sqrt{17}$
$\bullet^8$ pd find magnitude of a side	$\bullet^8$ $MN = \sqrt{21}$
$\bullet^9$ pd evaluate angle	$\bullet^9$ $76\cdot7^\circ$ or $1\cdot339$ rads or $85\cdot2$ grads

Notes

2. $\bullet^5$ is not available to candidates who choose to evaluate an incorrect angle.
3. For candidates who do not attempt $\bullet^9$, then $\bullet^5$ is only available if the formula quoted relates to the labelling in the question.
4. $\bullet^9$ should be awarded for any answer that rounds to 77° or $1\cdot3$ rads or 85 grads (i.e. correct to two significant figures.)

Regularly occurring responses

Response 1

$\cos M\hat{O}N = \dfrac{\overrightarrow{OM}.\overrightarrow{ON}}{\lvert\overrightarrow{OM}\rvert\lvert\overrightarrow{ON}\rvert}$ ✗ $\bullet^5$ Wrong angle

$\overrightarrow{OM}.\overrightarrow{ON} = 2$ ✗ $\bullet^6$

$\lvert\overrightarrow{OM}\rvert = 1$ ✗ $\bullet^7$ Eased because only one non-zero component.

$\lvert\overrightarrow{ON}\rvert = \sqrt{24}$ ✗ $\bullet^8$

$65\cdot9^\circ$ or $1\cdot150$ rads or $73\cdot2$ grads ✗ $\bullet^9$

3 marks out of 5

Response 2

$\cos M\hat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$ ✓ $\bullet^5$

$\overrightarrow{VM}.\overrightarrow{VN} = 0$ ✗ $\bullet^6$

$\lvert\overrightarrow{VM}\rvert = \sqrt{17}$ ✗ $\bullet^7$

$\lvert\overrightarrow{VN}\rvert = 2$ ✗ $\bullet^8$

90° or equivalent ✗ $\bullet^9$

Going directly to 90° from $\bullet^6$ would lose $\bullet^7$ and $\bullet^8$.

4 marks out of 5

2 (*a*) $12\cos x^\circ - 5\sin x^\circ$ can be expressed in the form $k\cos(x+a)^\circ$, where $k > 0$ and $0 \le a < 360$.
Calculate the values of *k* and *a*. **4**

Generic Scheme			Illustrative Scheme
(a)			
$\bullet^1$	ss	use addition formula	$\bullet^1$ $k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ or $k(\cos x^\circ \cos a^\circ - \sin x^\circ \cos a^\circ)$ *stated explicitly*
$\bullet^2$	ic	compare coefficients	$\bullet^2$ $k\cos a^\circ = 12$ and $k\sin a^\circ = 5$ or $-k\sin a^\circ = -5$ *stated explicitly*
$\bullet^3$	pd	process *k*	$\bullet^3$ 13 *no justification required, but do not accept* $\sqrt{169}$
$\bullet^4$	pd	process *a*	$\bullet^4$ $22\cdot6$ *accept any answer which rounds to* 23

Notes

1. Do not penalise the omission of the degree symbol.
2. Treat $k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$ as bad form only if the equations at the $\bullet^2$ stage both contain *k*.
3. $13\cos x^\circ \cos a^\circ - 13\sin x^\circ \sin a^\circ$ or $13(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x^\circ = 12$ and $k\sin x^\circ = 5$ or $-k\sin x^\circ = -5$, however, $\bullet^4$ is still available.
5. $\bullet^4$ is lost to candidates who give *a* in radians only.
6. $\bullet^4$ may be gained only as a consequence of using evidence at $\bullet^2$ stage.
7. Candidates may use any form of the wave equation for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however $\bullet^4$ is only available if the value of *a* is interpreted for the form $k\cos(x+a)^\circ$.

Regularly occurring responses

Response 1A

$k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$ ✓ $\bullet^1$

$\sin a = 5$ ✗ $\bullet^2$

$\cos a = 12$

$\tan a^\circ = \frac{5}{12}$

$a = 22\cdot6$ $\cancel{\checkmark}$ $\bullet^4$

$13\cos(x + 22\cdot6)$
✓ $\bullet^3$

2 marks out of 4

Response 1B

$k\cos x\cos a - k\sin x\sin a$ ✓ $\bullet^1$

$k = 13$ ✓ $\bullet^3$ ∧ $\bullet^2$

$\tan a^\circ = \frac{5}{12}$

$a = 22\cdot6$ $\cancel{\checkmark}$ $\bullet^4$

2 marks out of 4

Response 2

$k\cos(x - a)$

$= k\cos x\cos a + k\sin x\sin a$ $\cancel{\checkmark}$ $\bullet^1$

$= 13\cos x\cos a + 13\sin x\sin a$ ✓ $\bullet^3$

$13\cos a = 12$ $13\sin a = -5$ $\cancel{\checkmark}$ $\bullet^2$

then $a = 22\cdot6$ ✗ $\bullet^4$ See note 6

or $a = 337\cdot4$ ✗ $\bullet^4$ See note 7

3 marks out of 4

Response 3A

$k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$

$k\sin a = 5$ ✓ $\bullet^1$ ✓ $\bullet^2$

$k\cos a = 12$

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = \frac{12}{5}$ ✗ $\bullet^4$

$a = 67\cdot4$

3 marks out of 4

Response 3B

$k\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$

$k\sin a = 12$ ✓ $\bullet^1$ ✗ $\bullet^2$

$k\cos a = 5$

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = \frac{12}{5}$

$a = 67\cdot4$ $\cancel{\checkmark}$ $\bullet^4$

3 marks out of 4

Response 4

$k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ ✓ $\bullet^1$

$k\cos a = 12$ ✗ $\bullet^2$

$k\sin a = -5$

$k = 13$ ✓ $\bullet^3$ $\tan a^\circ = -\frac{5}{12}$

$a = 337\cdot4$ $\cancel{\checkmark}$ $\bullet^4$

See note 6

3 marks out of 4

2 (*b*) (i) Hence state the maximum and minimum values of $12\cos x^\circ - 5\sin x^\circ$.
(ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur. **3**

Generic Scheme			**Illustrative Scheme**
(b)			
$\bullet^5$	ss	state maximum and minimum	$\bullet^5$ 13, −13
$\bullet^6$	ic	find x corresponding to max. value	$\bullet^6$ maximum at $337\cdot4$ and no others
$\bullet^7$	pd	find x corresponding to min. value	$\bullet^7$ minimum at $157\cdot4$ and no others or $\bullet^6$ $337\cdot4$ and $157\cdot4$ and no others $\bullet^7$ maximum at $337\cdot4$ and minimum at $157\cdot4$

Notes

8. $\bullet^5$ is available for $\sqrt{169}$ and $-\sqrt{169}$ only if $\sqrt{169}$ has been penalised at $\bullet^3$.
9. Accept answers which round to 337 and 157 for $\bullet^6$ and $\bullet^7$.
10. Candidates who continue to work in radian measure should not be penalised further.
11. Extra solutions, correct or incorrect, should be penalised at $\bullet^6$ or $\bullet^7$ but not both.
12. $\bullet^6$ and $\bullet^7$ are not available to candidates who work with $13\cos(x+22\cdot6)^\circ = 0$ or $13\cos(x+22\cdot6)^\circ = 1$.
13. Candidates who use $13\cos(x-22\cdot6)^\circ$ from a correct (a) lose $\bullet^6$ but $\bullet^7$ is still available.

Regularly occurring responses

Response 1

From (a) $a = 67\cdot4$

max/ min $= \pm13$ ✓ $\bullet^5$

max at $292\cdot6$ ✔ $\bullet^6$

min at $112\cdot6$ ✔ $\bullet^7$

3 marks out of 3

Response 2

From (a) $\sqrt{169}\cos(x+22\cdot6)^\circ$

max $=\sqrt{169}$ min $=-\sqrt{169}$ ✔ $\bullet^5$

max at $22\cdot6$ ✗ $\bullet^6$

min at $202\cdot6$ ✔ $\bullet^7$

2 marks out of 3

$\sqrt{169}$ already penalised at (a)

Response 3A

$13\cos(x+22\cdot6)^\circ$ with $\updownarrow \times13$ and $\xrightarrow{} 22\cdot6$

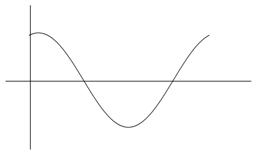
max at $22\cdot6$ ✗ $\bullet^6$

min at $202\cdot6$ ✔ $\bullet^7$

1 mark out of 3

Insufficient evidence for $\bullet^5$

Response 3B



$\xrightarrow[22\cdot6]{}$ $13\cos(x+22\cdot6)^\circ$

max at $22\cdot6$ ✗ $\bullet^6$

min at $202\cdot6$ ✔ $\bullet^7$

1 mark out of 3

N.B. Candidates who use differentiation in (b) can gain $\bullet^5$ only, as a direct result of their response in (a).
This question is in degrees and so calculus is not appropriate for $\bullet^6$ and $\bullet^7$.

3 (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**

Generic Scheme			Illustrative Scheme
(a)			
$\bullet^1$	ss	substitute	$\bullet^1$ $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
			Method 1 : Factorising
$\bullet^2$ $\bullet^3$	pd ic	express in standard form start proof	$\left.\begin{array}{l}\bullet^2\ \ 2x^2+4x+2\\ \bullet^3\ \ 2(x+1)(x+1)\end{array}\right\} = 0$ *see note* 1
$\bullet^4$	ic	complete proof	$\bullet^4$ equal roots so line is a tangent
			Method 2 : Discriminant
			$\bullet^2$ $2x^2 + 4x + 2 = 0$ *stated explicitly*
			$\bullet^3$ $4^2 - 4\times2\times2$
			$\bullet^4$ $b^2 - 4ac = 0$ so line is a tangent
$\bullet^5$	pd	coordinates of P	$\bullet^5$ $x = -1,\ y = 4$

Notes

For method 1 :

1. $\bullet^2$ is only available if "= 0" appears at either $\bullet^2$ or $\bullet^3$ stage.
2. Alternative wording for $\bullet^4$ could be e.g. 'repeated roots', 'repeated factor', ' only one solution', 'only one point of contact' **along with** 'line is a tangent'.

For both methods :

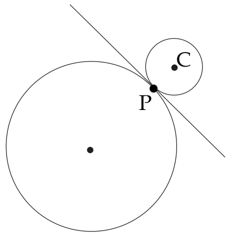
3. Candidates must work with a quadratic equation at the $\bullet^3$ and $\bullet^4$ stages.
4. Simply stating the tangency condition without supporting working cannot gain $\bullet^4$.
5. For candidates who obtain two distinct roots, $\bullet^4$ is still available for 'not equal roots so not a tangent' or '$b^2 - 4ac \neq 0$ so line is not a tangent', but $\bullet^5$ is not available.

3 (*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.

The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**



$x^2 + y^2 + 14x + 4y - 19 = 0$

Generic Scheme			Illustrative Scheme	
(b)				
Method 1 : via centre and radius			**Method 1** : via centre and radius	
•6	ic	state centre of larger circle	$(-7, -2)$	*see note* 11
•7	ss	find radius of larger circle	$\sqrt{72}$	*see note* 6 *stated, or implied by* •8
•8	pd	find radius of smaller circle	$\sqrt{8}$	*see note* 7
•9	ss	strategy for finding centre	e.g "Stepping out"	
•10	ic	interpret centre of smaller circle	(1, 6)	
•11	ic	state equation	$(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$	
Method 2 : via ratios			**Method 2** : via ratios	
•6	ic	state centre of larger circle	$(-7, -2)$	*see note* 11
•7	ss	strategy for finding centre	e.g. "Stepping out"	
•8	ic	state centre of smaller circle	(1, 6)	
•9	ss	strategy for finding radius	$\sqrt{2^2 + 2^2}$	
•10	pd	find radius of smaller circle	$\sqrt{8}$	*see note* 10
•11	ic	state equation	$(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$	

Notes

For method 1:

6. Acceptable alternatives for •7 are $6\sqrt{2}$ or decimal equivalent which rounds to $8 \cdot 5$ i.e. to two significant figures.
7. Acceptable alternatives for •8 are $\frac{\sqrt{72}}{3}$ or $2\sqrt{2}$ or decimal equivalent which rounds to $2 \cdot 8$.
8. (1, 6) without working gains •10 but loses •9.

For method 2:

9. (1, 6) without working gains •8 but loses •7.
10. Acceptable alternatives for •10 are $2\sqrt{2}$ or decimal equivalent which rounds to $2 \cdot 8$.

In both methods:

11. If $m = 1$ is used in a 'stepping out' method the centre of the larger circle need not be stated explicitly for •6.
12. For the smaller circle, candidates who 'guess' values for either the centre or radius cannot be awarded •11.
13. At •11 e.g. $\sqrt{8}^2$, $2 \cdot 8^2$ are unacceptable, but any decimal which rounds to $7 \cdot 8$ is acceptable.
14. •11 is not available to candidates who divide the coordinates of the centre of the larger circle by 3.

4 Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$. **5**

Generic Scheme			Illustrative Scheme
4			
			Method 1 : Using factorisation
$\bullet^1$	ss	know to use double angle formula	$\bullet^1$ $2\times(2\cos^2 x - 1\,)\ldots$
$\bullet^2$	ic	express as quadratic in $\cos x$	$\bullet^2$ $4\cos^2 x - 5\cos x - 6$
$\bullet^3$	ss	start to solve	$\bullet^3$ $(4\cos x + 3)(\cos x - 2)$ = 0 must appear at either of these lines to gain $\bullet^2$.
			Method 2 : Using quadratic formula
			$\bullet^1$ $2\times(2\cos^2 x - 1)\ldots$
			$\bullet^2$ $4\cos^2 x - 5\cos x - 6 = 0$
			$\bullet^3$ $\cos x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4\times 4\times(-6)}}{2\times 4}$
			In both methods :
$\bullet^4$	pd	reduce to equations in $\cos x$ only	$\bullet^4$ $\cos x = -\frac{3}{4}$ and $\cos x = 2$
$\bullet^5$	pd	complete solutions to include only one where $\cos x = k$ with $\lvert k\rvert > 1$	$\bullet^5$ $2\cdot419,\ 3\cdot864$ and no solution
			or
			$\bullet^4$ $\cos x = 2$ and no solution
			$\bullet^5$ $\cos x = -\frac{3}{4}$ and $2\cdot419,\ 3\cdot864$

Notes

1. $\bullet^1$ is not available for simply stating that $\cos 2\mathrm{A} = 2\cos^2 \mathrm{A} - 1$ with no further working.
2. Substituting $\cos 2\mathrm{A} = 2\cos^2 \mathrm{A} - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc. should be treated as bad form throughout.
3. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, $\bullet^1$ cannot be given until the equation reduces to a quadratic in $\cos x$.
4. Candidates may express the quadratic equation obtained at the $\bullet^2$ stage in the form $4c^2 - 5c + 6 = 0$, $4x^2 - 5x + 6 = 0$ etc. For candidates who do not solve a trig. equation at $\bullet^5$, $\cos x$ must appear explicitly to gain $\bullet^4$.
5. $\bullet^4$ and $\bullet^5$ are only available as a consequence of solving a quadratic equation subsequent to a substitution.
6. Any attempt to solve $a\cos^2 x + b\cos x = c$ loses $\bullet^3$, $\bullet^4$ and $\bullet^5$.
7. Accept answers given as decimals which round to $2\cdot4$ and $3\cdot9$.
8. There must be an indication after $\cos x = 2$ that there are no solutions to this equation.

   Acceptable evidence : e.g. "$\cancel{\cos x = 2}$" , "NA", "out of range", "invalid" and "$\cos x = 2$ no", "$\cos x = 2$ ✗"

   Unacceptable evidence : e.g. "$\underline{\underline{\cos x = 2}}$", "$\cos x = 2$???", "Maths Error".
9. $\bullet^5$ is not available to candidates who work throughout in degrees and do not convert their answer into radian measure.
10. Do not accept e.g. $221\cdot4,\ 138\cdot6,\ \dfrac{221\cdot4\pi}{180},\ \dfrac{221\pi}{180},\ 1\cdot23\pi$.
11. Ignore correct solution outside the interval $0 \le x < 2\pi$.

4

Regularly occurring responses

Response 1

$2\times 2\cos^2 x-1\ldots$ ✓ $\bullet^1$

$4\cos^2 x-5\cos x-5=0$ ✗ $\bullet^2$

$\cos x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4\times 4\times(-5)}}{2\times 4}$ ✗✓ $\bullet^3$

$\cos x=\frac{5-\sqrt{105}}{8}$ and $\cos x=\frac{5+\sqrt{105}}{8}$ ✗✓ $\bullet^4$

$2\cdot 286,\ 3\cdot 997$ and no solution ✗✓ $\bullet^5$

$\bullet^5$ is only available to candidates where one, but not both, of their equations has no solution for $\cos x$.

4 marks out of 5

Response 2

$4\cos^2 x-1\ldots$ ✓ $\bullet^1$

$4\cos^2 x-5\cos x-5=0$ ✗ $\bullet^2$

$(2\cos x+1)(2\cos x-5)=0$ ✗ $\bullet^3$

$\cos x=-\frac{1}{2}$ and $\cancel{\cos x=\frac{5}{2}}$ ✗✓ $\bullet^4$

$x=\frac{2\pi}{3},\ \frac{4\pi}{3}$ ✗✓ $\bullet^5$

$4\cos^2 x-1$ with no further working cannot gain $\bullet^1$; however if a quadratic in $\cos x$ subsequently appears then $\bullet^1$ is awarded but $\bullet^2$ is not available.

3 marks out of 5

$\bullet^1$ is lost here as it is not clear whether the candidate has used $2\cos^2 x-1$ or $\cos^2 x-1$ as their substitution.

Response 3A

$2\cos^2 x-1-5\cos x-4=0$ ✗ $\bullet^1$

$2\cos^2 x-5\cos x-5=0$ ✗✓ $\bullet^2$

$\cos x=\dfrac{5\pm\sqrt{25+40}}{4}$ ✗✓ $\bullet^3$

$\cos x=-0\cdot 766$ and $\cos x=3\cdot 267$ ✗✓ $\bullet^4$

$x=2\cdot 44,\ 3\cdot 84$ and undefined ✗✓ $\bullet^5$

4 marks out of 5

Response 3B

$\cos 2x=2\cos^2 x-1$

$2\cos^2 x-1-5\cos x-4=0$ ✓ $\bullet^1$

$2\cos^2 x-5\cos x-5=0$ ✗ $\bullet^2$

$\cos x=\dfrac{5\pm\sqrt{25+40}}{4}$ ✗✓ $\bullet^3$

$\cos x=-0\cdot 766$ and $\cos x=3\cdot 267$ ✗✓ $\bullet^4$

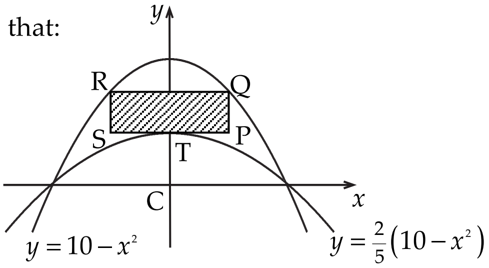
$x=2\cdot 44,\ 3\cdot 84$ and undefined ✗✓ $\bullet^5$

4 marks out of 5

5 The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola.
- RQ and SP are parallel to the x-axis.
- T , the turning point of the lower parabola, lies on SP.



(*a*) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3$$

3

(*b*) Find the maximum area of this rectangle. **6**

Generic Scheme **Illustrative Scheme**

5 (a)

$\bullet^1$	ss	know to and find OT	$\bullet^1$	4 or $(0,4)$	*stated, or implied by* $\bullet^2$
$\bullet^2$	ic	obtain an expression for PQ	$\bullet^2$	$10 - x^2 - 4$	
$\bullet^3$	ic	complete area evaluation	$\bullet^3$	$2x \times (6 - x^2) = 12x - 2x^3$	

Notes

1. The evidence for $\bullet^1$ and $\bullet^2$ may appear on a sketch.
2. No marks are available to candidates who work backwards from the area formula.
3. $\bullet^3$ is only available if $\bullet^2$ has been awarded.

5 (b)

$\bullet^4$	ss	know to and start to differentiate	$\bullet^4$	$A'(x) = 12\ ...$	*stated, or implied by* $\bullet^5$
$\bullet^5$	pd	complete differentiation	$\bullet^5$	$12 - 6x^2$	
$\bullet^6$	ic	set derivative to zero	$\bullet^6$	$12 - 6x^2 = 0$	
$\bullet^7$	pd	obtain x	$\bullet^7$	$\sqrt{2}$ or decimal equivalent $\left(\text{ignore inclusion of } -\sqrt{2}\right)$	
$\bullet^8$	ss	justify nature of stationary point	$\bullet^8$		

x	...	$\sqrt{2}$	...
$A'(x)$	+	0	−

(Note : accept $12 - 6x^2$ in lieu of $A'(x)$ in the nature table.)

$\bullet^9$	ic	interpret result and evaluate area	$\bullet^9$	Max **and** $8\sqrt{2}$ or decimal equivalent N.B. To conclude a maximum the evidence must come from $\bullet^8$.

Notes

4. At $\bullet^7$ accept any answer which rounds to $1\cdot4$.
5. Throughout this question treat the use of $f'(x)$ or $\frac{dy}{dx}$ as bad form.
6. At $\bullet^8$ the nature can be determined using the second derivative.
7. At $\bullet^9$ accept any answer which rounds to $11\cdot3$ or $11\cdot4$.

5

Regularly occurring responses

Response 1

$A(x) = 12x - 2x^3$

$A'(x) = 24x^2 - 6x^3$ ✗ $\bullet^4$ ✗ $\bullet^5$

$24x^2 - 6x^3 = 0$ $\cancel{\checkmark}$ $\bullet^6$

$A'(x) = 0$ on its own would not be sufficient for $\bullet^6$.

Response 2

At stationary points, $A'(x) = 0$

$12 - 6x^2$ ✓ $\bullet^4$ ✓ $\bullet^5$ ✓ $\bullet^6$

$x = \sqrt{2}$ ✓ $\bullet^7$

Response 3

$A(x) = 12x - 2x^3$

$= 12 - 6x^2$ ✓ $\bullet^4$ ✓ $\bullet^5$

Bad form

Response 4A

	$\dots$	$\sqrt{2}$	$\dots$
$A'(x)$	+	0	–

$\cancel{\checkmark}$ $\bullet^8$

x missing

Response 4B

x	1	$\sqrt{2}$	2
$A'(x)$	6	0	–12

✓ $\bullet^8$

Response 4C

x	$\dots$	$\sqrt{2}$	$\dots$
$A'(x)$	╱	─	╲

$\cancel{\checkmark}$ $\bullet^8$

or 'slope' etc.

signs or values are necessary

Response 5

Maximum at $x = \sqrt{2}$

$y = 12\sqrt{2} - 2\sqrt{2}^3 = 8\sqrt{2}$ ✓ $\bullet^9$

Area $= 2\sqrt{2} \times 8\sqrt{2} = 32$

Treat this as an error subsequent to a correct answer.

6 (*a*) A curve has equation $y=(2x-9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x=9$ is $y=\frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

Find the coordinates of point A. **1**

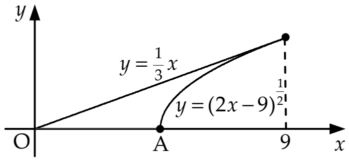


Diagram 1

		Generic Scheme	Illustrative Scheme
6 (a)			
$\bullet^1$	ss	know to and start to differentiate	$\bullet^1$ $\frac{1}{2}(2x-9)^{-\frac{1}{2}}$
$\bullet^2$	pd	complete chain rule derivative	$\bullet^2$ $\ldots\times 2$
$\bullet^3$	pd	gradient via differentiation	$\bullet^3$ $\frac{1}{3}$
$\bullet^4$	pd	obtain y_{CURVE} at $x=9$	$\bullet^4$ 3
$\bullet^5$	ic	state equation and complete	$\bullet^5$ $y-3=\frac{1}{3}(x-9)$ and complete to $y=\frac{1}{3}x$

Notes

1. $\bullet^3$ is only available as a consequence of differentiating equation of the curve.
2. Candidates must arrive at the equation of the tangent via the point $(9, 3)$ and not the origin.
3. For $\bullet^3$ accept $9^{-\frac{1}{2}}$.

Regularly occurring responses

Response 1
Candidates who equate derivatives:

$$\underset{\checkmark\,\bullet^1}{\frac{1}{2}(2x-9)^{-\frac{1}{2}}}\underset{\checkmark\,\bullet^2}{\times 2}=\frac{1}{3} \quad \checkmark\,\bullet^3$$

leading to $x=9$ and $y=3$ from curve ✓ $\bullet^4$
Also obtaining $y=3$ from line and so line is a tangent ✓ $\bullet^5$

5 marks out of 5

Response 2
Candidates who intersect curve and line:

$$(2x-9)^{\frac{1}{2}}=\frac{1}{3}x \quad \checkmark\,\bullet^1$$

$$2x-9=\left(\frac{1}{3}x\right)^2 \quad \checkmark\,\bullet^2$$

$$\frac{1}{9}x^2-2x+9=0 \quad \checkmark\,\bullet^3$$

5 marks out of 5

Factorising or using discriminant ✓ $\bullet^4$
Equal roots or $b^2-4ac=0$ so line is a tangent ✓ $\bullet^5$

(b)			
$\bullet^6$	ic	obtain coordinates of A	$\bullet^6$ $\left(\frac{9}{2}, 0\right)$

Notes

4. Accept $x=\frac{9}{2}$, $y=0$ where $y=0$ may appear from $(2x-9)^{\frac{1}{2}}=0$.
5. For $\left(\frac{9}{2}, 0\right)$ without working $\bullet^6$ is awarded, but from erroneous working $\bullet^6$ is lost (see response below).

Regularly occurring responses

Response 1

$$\sqrt{2x-9}=0 \;\Rightarrow\; \sqrt{2x}-3=0 \;\Rightarrow\; 2x-9=0 \;\Rightarrow\; x=4\cdot 5$$

Here $\bullet^6$ cannot be awarded due to the erroneous working.

6 *(c)* Calculate the shaded area shown in diagram 2. **7**

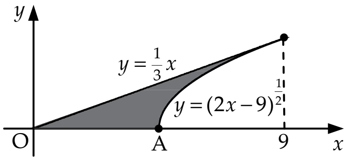


Diagram 2

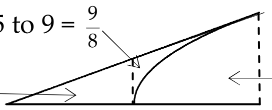
Generic Scheme	Illustrative Scheme
6 (c)	
Method 1 : Area of triangle – area under curve	**Method 1** : Area of triangle – area under curve
•⁷ ss strategy for finding shaded area	•⁷ Shaded area = Area of large Δ − Area under curve
•⁸ ss know to integrate $\left(2x-9\right)^{\frac{1}{2}}$	•⁸ $\int(2x-9)^{\frac{1}{2}}\,dx$
•⁹ pd start integration	•⁹ $\frac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
•¹⁰ pd complete integration	•¹⁰ $...\times\frac{1}{2}$
•¹¹ ic limits x_A and 9	•¹¹ $\frac{9}{2}$ and 9
•¹² pd substitute limits	•¹² $\frac{1}{3}(18-9)^{\frac{3}{2}}-0$
•¹³ pd evaluate area and complete strategy	•¹³ $\frac{27}{2}-9=\frac{9}{2}$ or $4\frac{1}{2}$ or $4\cdot5$
Method 2 : Area between line and curve	**Method 2** : Area between line and curve
•⁷ ss strategy for finding shaded area	•⁷ Area of small Δ + area between line and curve
•⁸ ss know to integrate $\left(2x-9\right)^{\frac{1}{2}}$	•⁸ $\int ...\,(2x-9)^{\frac{1}{2}}\;dx$
•⁹ pd start integration	•⁹ $...\frac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
•¹⁰ pd complete integration	•¹⁰ $...\times\frac{1}{2}$
•¹¹ ic limits x_A and 9	•¹¹ $\frac{9}{2}$ and 9
•¹² pd 'upper − lower' and substitute limits	•¹² $\left(\frac{1}{6}\times9^2-\frac{1}{3}(18-9)^{\frac{3}{2}}\right)-\left(\frac{1}{6}\times\left(\frac{9}{2}\right)^2-\frac{1}{3}(9-9)^{\frac{3}{2}}\right)$
•¹³ pd evaluate area and complete strategy	•¹³ $\frac{27}{8}+\frac{9}{8}=\frac{9}{2}$ or $4\frac{1}{2}$ or $4\cdot5$

Notes

6. •⁷ may not be obvious until the final line of working and may be implied by final answer or a diagram.
7. At •¹¹ the value of x_A must lie between 0 and 9 exclusively, however, •¹² and •¹³ are only available if $4\cdot5\le x_A<9$.
8. Full marks are available to candidates who integrate with respect to y.

- -

You may find the following helpful in marking this question:



Area of $\Delta_{\text{LARGER}}=13\cdot5$ or $\frac{27}{2}$

Area between curve and line from 4·5 to 9 = $\frac{9}{8}$

Area under curve from 4·5 to 9 = 9

Area of $\Delta_{\text{SMALLER}}=\frac{27}{8}$

7 (*a*) Given that $\log_4 x = \text{P}$, show that $\log_{16} x = \frac{1}{2}\text{P}$. **3**

Generic Scheme			**Illustrative Scheme**	
(a)				
$\bullet^1$	ss	convert from log to exponential form	$\bullet^1$	$x = 4^{\text{P}}$
$\bullet^2$	ss	know to and convert back to log form	$\bullet^2$	$\log_{16} x = \log_{16} 4^{\text{P}}$
$\bullet^3$	pd	process and complete	$\bullet^3$	$\log_{16} x = \text{P} \times \log_{16} 4$ and complete

Notes

1. No marks are available to candidates who simply substitute in values and verify the result.

   e.g. $\log_4 4 = 1$ and $\log_{16} 4 = \frac{1}{2}$ ✗

   $\log_4 x = \text{P}$ and $\log_{16} x = \frac{1}{2}\text{P}$

Regularly occurring responses

Response 1

$\log_4 x = \text{P}$

$x = 4^{\text{P}}$ ✓ $\bullet^1$

$\log_{16} x = \frac{1}{2}P$ ✗ $\bullet^2$

$x = 16^{\frac{1}{2}\text{P}}$

$= 4^{\text{P}}$ $\wedge$ $\bullet^3$

1 mark out of 3

Response 2

$\log_4 x = P$

$x = 4^P$ ✓ $\bullet^1$

$x^2 = 4^{2P}$

$= 16^P$ ✗ $\bullet^2$

$\log_{16} x^2 = P$

$2\log_{16} x = P$

$\log_{16} x = \frac{1}{2}P$ ✓ $\bullet^3$

2 marks out of 3

Response 3

$x = 4^{\text{P}}$ ✓ $\bullet^1$

$x = \left(16^{\frac{1}{2}}\right)^{\text{P}}$ or $16^{\frac{1}{2}\times\text{P}}$ ✓ $\bullet^2$

$x = 16^{\frac{1}{2}\text{P}}$

$\log_{16} x = \frac{1}{2}\text{P}$ ✓ $\bullet^3$

Without this step $\bullet^2$ would be lost but $\bullet^3$ is still available as follow through.

3 marks out of 3

Response 4

$x = 4^{\text{P}}$ ✓ $\bullet^1$

$\log_4 x = \text{P}$

$4^{\log_4 x} = 4^{\text{P}} = x$

$4^{\log_4 x} = x$

$\log_{16} x = k\text{P}$

$16^{\log_{16} x} = 16^{k\text{P}}$

$x = 16^{k\text{P}}$

$4^{\text{P}} = 16^{k\text{P}}$

$4 = 16^k$ ✗ $\bullet^2$

$k = \frac{1}{2}$

$\therefore \log_{16} x = \frac{1}{2}\text{P}$ as $16^{\frac{1}{2}} = 4$ ✓ $\bullet^3$

2 marks out of 3

Response 5

Beware that some candidates give a circular argument.

This is only worth $\bullet^1$.

$\log_4 x = \text{P}$ then $\log_{16} x = \frac{1}{2}\text{P}$

$x = 4^{\text{P}}$ ✓ $\bullet^1$ $\qquad$ $x = 16^{\frac{1}{2}\text{P}}$

$\log_4 x = \log_4 4^{\text{P}}$ $\qquad$ $\log_{16} x = \log_{16} 16^{\frac{1}{2}\text{P}}$

$\log_4 x = \text{P}\log_4 4$ $\qquad$ $\log_{16} x = \frac{1}{2}\text{P}\log_{16} 16$

$\log_4 x = \text{P}$ $\qquad$ $\log_{16} x = \frac{1}{2}\text{P}$ ✗

1 mark out of 3

Response 6

$$\log_{16} x = \frac{\log_4 x}{\log_4 16} = \frac{\log_4 x}{2} = \frac{1}{2}\text{P}$$

✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$

Using change of base result.

3 mark out of 3

7 (*b*) Solve $\log_3 x + \log_9 x = 12.$ **3**

Generic Scheme			Illustrative Scheme	
(b)				
$\bullet^4$	ss	use appropriate strategy	$\bullet^4$ $\log_3 x + \frac{1}{2}\log_3 x = 12$	$\bullet^4$ $2\log_9 x + \log_9 x = 12$
$\bullet^5$	pd	start solving process	$\bullet^5$ $\log_3 x = 8$	$\bullet^5$ $\log_9 x = 4$
$\bullet^6$	pd	complete process via log to expo form	$\bullet^6$ $x = 3^8 \quad (= 6561)$	$\bullet^6$ $x = 9^4 \quad (= 6561)$
			or	**or**
			$\bullet^4$ $Q + \frac{1}{2}Q = 12$ $Q = 8$	$\bullet^4$ $2Q + Q = 12$ $Q = 4$
			$\bullet^5$ $\log_3 x = 8$	$\bullet^5$ $\log_9 x = 4$
			$\bullet^6$ $x = 3^8 \quad (= 6561)$	$\bullet^6$ $x = 9^4 \quad (= 6561)$

Notes

2. At $\bullet^4$ any letter except x may be used in lieu of Q.
3. Candidates who use a trial and improvement technique by substituting values for x gain no marks.
4. The answer with no working gains no marks.

Regularly occurring responses

Response 1

$Q + 2Q = 12$

$3Q = 12$

$Q = 4$

$\log_3 x = 4$

$x = 3^4$ ✓✗ $\bullet^6$

$= 81$

✓ $\bullet^4$ / ✗ $\bullet^5$ or ✗ $\bullet^4$ / ✓✗ $\bullet^5$

2 marks out of 3

The marks allocated are dependent on what substitution is used for Q.

Response 2

$\log_3 x + 2\log_3 x = 12$ ✗ $\bullet^4$

$3\log_3 x = 12$

$\log_3 x = 4$ ✓✗ $\bullet^5$

$x = 3^4$ ✓✗ $\bullet^6$

$= 81$

2 marks out of 3

Response 3

$2\log_9 x + \log_9 x = 12$ ✓ $\bullet^4$

$\log_9 x^2 + \log_9 x = 12$

$\log_9 x^3 = 12$ ✓ $\bullet^5$

$x^3 = 9^{12}$

$x = \sqrt[3]{9^{12}}$

$x = 9^4$ ✓ $\bullet^6$

$x = 3^8$

$= 6561$

3 marks out of 3

Response 4

$\log_3 x = 8$ ✓✓✗ $\bullet^4$ ✓✓✗ $\bullet^5$

$x = 3^8$ ✓ $\bullet^6$

$= 6561$

Without justification, $\bullet^4$ and $\bullet^5$ are not available.