Paper 1 Section A

Question	Answer
1	C
2	D
3	B
4	B
5	A
6	C
7	A
8	C
9	A
10	B
11	D
12	B
13	D
14	A
15	D
16	C
17	D
18	B
19	B
20	A

Summary		
	A	5
	B	6
	C	4
	D	5

21	(a)	(i)	Show that $(x-4)$ is a factor of x^3-5x^2+2x+8.	
		(ii)	Factorise x^3-5x^2+2x+8 fully.	
		(iii)	Solve $x^3-5x^2+2x+8=0$.	6

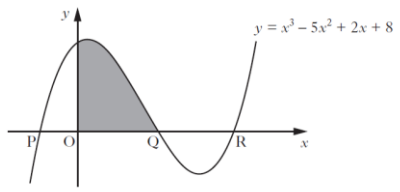
Generic Scheme | **Illustrative Scheme**

21 (a)

Generic Scheme			Illustrative Scheme
			Method 1 : Using synthetic division
•1	ss	know to use $x=4$	•1 $\begin{array}{c\|cccc} 4 & 1 & -5 & 2 & 8 \\ & & & & \\ \hline \end{array}$
•2	pd	complete evaluation	•2 $\begin{array}{c\|cccc} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$
•3	ic	state conclusion	•3 'remainder is zero so $(x-4)$ is a factor'
•4	ic	find quadratic factor	•4 x^2-x-2 **stated, or implied by •5**
•5	pd	factorise completely	•5 $(x-4)(x-2)(x+1)$ **stated explicitly in any order**
•6	ic	state solutions	•6 $-1,\ 2,\ 4$
			Method 2 : Using substitution and inspection
			•1 know to use $x=4$
			•2 $64-80+8+8=0$
			•3 $(x-4)$ is a factor
			•4 $(x-4)(x^2-x-2)$ **stated, or implied by •5**
			•5 $(x-4)(x-2)(x+1)$ **stated explicitly in any order**
			•6 $-1,\ 2,\ 4$
			6

Notes

1. •3 is only available as a consequence of the evidence for •1 and •2.
2. Communication at •3 must be consistent with working at •2.
   i.e. candidate's working must arrive legitimately at zero before •3 is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-4)$ is not a factor'.
3. Accept any of the following for •3:
   - '$f(4)=0$ so $(x-4)$ is a factor'
   - 'since remainder is 0, it is a factor'
   - the 0 from table linked to word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'.
4. Do not accept any of the following for •3:
   - double underlining the zero or boxing in the zero, without a comment
   - '$x=4$ is a factor', '$(x+4)$ is a factor', '$x=4$ is a root', '$(x-4)$ is a root'
   - the word 'factor' **only**, with no link.
5. To gain •6, $4, -1, 2$ **must** appear together in (a).
6. $(x-4)(x-2)(x+1)$ leading to $(4,\ 0)$, $(2,\ 0)$ and $(-1,\ 0)$ **only** does not gain •6.
7. $(x-2)(x+1)$ only, leading to $x=2, x=-1$ does not gain •6 as equation solved is not a cubic.
8. Candidates who attempt to solve the cubic equation subsequent to $x=-1, 2, 4$ and obtain different solutions, or no solutions, cannot gain •6.

21 (b)	The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$. The curve crosses the x-axis at P, Q and R. Determine the shaded area. 	6

Generic Scheme | **Illustrative Scheme**

21 (b)

		Generic Scheme	Illustrative Scheme
•7	ic	identify x_Q from working in (a)	2
•8	ic	interpret appropriate limits	0, 2
•9	ss	know and start to integrate	integrate one term correctly (but see Note 10)
•10	pd	complete integration	$\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x$ or equivalent
•11	ic	substitute limits	$\left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8\times 2\right) - 0$
•12	pd	state area	$\frac{32}{3}$ or $10\frac{2}{3}$ but not a decimal approximation

6

Notes

9. Evidence for •7 and •8 may not appear until •11 stage.
10. Where a candidate differentiates one or more terms at •9, then •9, •10, •11 and •12 are not available.
11. Candidates who substitute at •11, without integrating at •9, do not gain •9, •10, •11 and •12.
12. For candidates who make an error in (a), •8 is only available if 0 is the lower limit and a positive integer value is used for the upper limit.
13. •11 is only available where both limits are numerical values.
14. Candidates must show evidence that they have considered the lower limit 0 in their substitution at •11 stage.

Regularly occurring responses

Response 1

Candidates who use Q throughout

Candidate A

$\int_0^Q (x^3 - 5x^2 + 2x + 8)\,dx$ •7 ✗ •8 ✗

$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x\right]_0^Q$ •9 ✓ •10 ✓

$= \frac{1}{4}Q^4 - \frac{5}{3}Q^3 + Q^2 + 8Q - 0$ •11 ✗ •12 ✗

However, if Q is replaced by 2 at this stage, and working continues, all 6 marks may still be available .

Response 2

Dealing with negatives

Candidate B

Q(−1, 0) ✗ •7

$\int_0^{-1} (x^3 - 5x^2 + 2x + 8)\,dx$ ✗ •8

$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_0^{-1}$ ✓ •9 ✓ •10

$= \frac{1}{4}(-1)^4 - \frac{5}{3}(-1)^3 + (-1)^2 + 8(-1) - 0$ ✗ •11

$= -\frac{61}{12}$

cannot be negative so $\frac{61}{12}$ ✗ •12

but

$= -\frac{61}{12}$

$A = \frac{61}{12}$ ✗ •12

22 (a)	The expression $\cos x-\sqrt{3}\sin x$ can be written in the form $k\cos(x+a)$ where $k>0$ and $0\le a<2\pi$. Calculate the values of k and a.	4

Generic Scheme | **Illustrative Scheme**

22 (a)

	Generic Scheme		Illustrative Scheme	
•1	ss	use compound angle formula	$k\cos x\cos a - k\sin x\sin a$	**stated explicitly**
•2	ic	compare coefficients	$k\cos a = 1$ and $k\sin a=\sqrt{3}$	**stated explicitly**
•3	pd	process k	2 (do not accept $\sqrt{4}$)	
•4	pd	process a	$\frac{\pi}{3}$ **but** must be consistent with •2	

4

Notes

1. Treat $k\cos x\cos a-\sin x\sin a$ as bad form only if the equations at the •2 stage both contain k.
2. $2\cos x\cos a-2\sin x\sin a$ or $2(\cos x\cos a-\sin x\sin a)$ is acceptable for •1 and •3.
3. Accept $k\cos a=1$ and $-k\sin a=-\sqrt{3}$ for •2.
4. •2 is not available for $k\cos x=1$ and $k\sin x=\sqrt{3}$, however, •4 is still available.
5. •4 is only available for a single value of a.
6. Candidates who work in degrees and do not convert to radian measure in (a) do not gain •4.
7. Candidates may use any form of the wave equation for •1, •2 and •3, however, •4 is only available if the value of a is interpreted for the form $k\cos(x+a)$.

Regularly occurring responses

Response 1 : Missing information in working

Candidate A

$2\cos a=1$
$-2\sin a=-\sqrt{3}$ ✓
$\tan a=\frac{\sqrt{3}}{1}$
$a=\frac{\pi}{3}$ ✓

•1 ✗
•2 ✓
•3 ✓
•4 ✓

3 marks out of 4

Candidate B

$\cos a=1$
$\sin a=\sqrt{3}$
$\tan a=\frac{\sqrt{3}}{1}$
$a=\frac{\pi}{3}$

•1 ✗
•2 ✗
•3 ✗
•4 ✗

0 marks out of 4

Not consistent with evidence at •2.

Response 2 : Correct expansion of $k\cos(x+a)$ and possible errors for •2 and •4

Candidate C

$k\cos a=1$
$k\sin a=\sqrt{3}$ ✓ •2
$\tan a=\frac{1}{\sqrt{3}}$ so $a=\frac{\pi}{6}$ ✗ •4

Candidate D

$k\cos a=\sqrt{3}$ ✗ •2
$k\sin a=1$
$\tan a=\frac{1}{\sqrt{3}}$ so $a=\frac{\pi}{6}$ ✗ •4

Candidate E

$k\cos a=1$
$k\sin a=-\sqrt{3}$ ✗ •2
$\tan a=-\sqrt{3}$ so $a=\frac{5\pi}{3}$ ✗ •4

Response 3 : Labelling incorrect using $\cos(A+B)=\cos A\cos B-\sin A\sin B$ from formula list

Candidate F

$k\cos A\cos B-k\sin A\sin B$ ✗ •1
$k\cos a=1$
$k\sin a=\sqrt{3}$ ✓ •2
$\tan a=\sqrt{3}$ so $a=\frac{\pi}{3}$ ✓ •4

Candidate G

$k\cos A\cos B-k\sin A\sin B$ ✗ •1
$k\cos x=1$ ✗ •2
$k\sin x=\sqrt{3}$
$\tan x=\sqrt{3}$ so $x=\frac{\pi}{3}$ ✗ •4

Candidate H

$k\cos A\cos B-k\sin A\sin B$ ✗ •1
$k\cos B=1$
$k\sin B=\sqrt{3}$ ✗ •2
$\tan B=\sqrt{3}$ so $B=\frac{\pi}{3}$ ✗ •4

22 (b) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \le x \le 2\pi$. **3**

Generic Scheme			Illustrative Scheme
22 (b)			
$\bullet^5$	ic	interpret y-intercept	$\bullet^5$ 1
$\bullet^6$	ss	strategy for finding roots	$\bullet^6$ e.g. $2\cos\left(x+\frac{\pi}{3}\right)=0$ **or** $\sqrt{3}\sin x = \cos x$
$\bullet^7$	ic	state both roots	$\bullet^7$ $\frac{\pi}{6}$, $\frac{7\pi}{6}$

3

Notes

8. Candidates should only be penalised once for leaving their answer in degrees in (a) and (b).
9. If the expression used in (b) is not consistent with (a) then only $\bullet^5$ and $\bullet^7$ are available.
10. Correct roots without working cannot gain $\bullet^6$ but will gain $\bullet^7$.
11. Candidates should only be penalised once for not simplifying $\sqrt{4}$ in (a) and (b).

Regularly occurring responses

Response 4 : Communication for $\bullet^5$

Candidate I

(1, 0) without working. ✗ $\bullet^5$

Candidate J

$\cos 0 - \sqrt{3}\sin 0 = 1$ ✓ $\bullet^5$
so (1, 0).

Response 5 : Follow through from a wrong value of a

Candidate K

From (a) $a = \frac{\pi}{6}$ — $\bullet^6$ ✗

then in (b) $x = \frac{\pi}{3}$, $\frac{4\pi}{3}$ only — $\bullet^7$ ✓

Candidate L

From (a) $a = 60^\circ$ ✗ $\bullet^4$ — $\bullet^6$ ✗

then in (b) $x = 30^\circ$, 210° only — $\bullet^7$ ✓

Note 10

Response 6 : Root or graphical approach

Candidate M

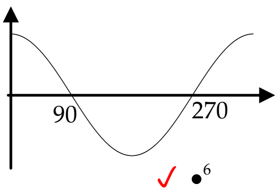
$\frac{\pi}{2} - \frac{\pi}{3}$ and $\frac{3\pi}{2} - \frac{\pi}{3}$ ✓ $\bullet^6$

$= \frac{\pi}{6}$ and $\frac{7\pi}{6}$ ✓ $\bullet^7$

Candidate N

(a) 60° ✗ $\bullet^4$

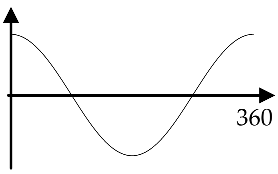
(b)



✓ $\bullet^6$

When $x = 30^\circ, 210^\circ$ ✗ $\bullet^7$

Candidate O



moved 60° to left ✓ $\bullet^6$

cuts x-axis at $\frac{\pi}{6}$, $\frac{2\pi}{3}$ ✗ $\bullet^7$

Response 7 : Circular argument not leading anywhere

Candidate P

$2\cos x \times \frac{1}{2} - 2\sin x \times \frac{\sqrt{3}}{2} = 0$ — $\bullet^6$ ✗

$\cos x - \sqrt{3}\sin x = 0$ — $\bullet^7$ ✗

Response 8 : Transcription error in (b)

Candidate Q

(a) correct

(b) $2\cos\left(x - \frac{\pi}{3}\right) = 0$ so $x = \frac{5\pi}{6}$, $\frac{11\pi}{6}$ — ✗ $\bullet^6$ ✓ $\bullet^7$

$y = 2\cos\left(0 - \frac{\pi}{3}\right) = 2\cos\left(-\frac{\pi}{3}\right) = 1$ ✓ $\bullet^5$

$x - \frac{\pi}{3}$ is penalised as $x + \frac{\pi}{3}$ obtained in (a).
However $\bullet^5$ and $\bullet^7$ are still available as follow through. See Note 9.

23 (a) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, −3) to Q(−1, 9).	4

Generic Scheme | **Illustrative Scheme**

23 (a)

•[1]	ss	find midpoint of PQ	•[1]	(1, 3)
•[2]	ss	find gradient of PQ	•[2]	−3
•[3]	ic	interpret perpendicular gradient	•[3]	$\frac{1}{3}$
•[4]	ic	state equation of perp. bisector	•[4]	$y-3=\frac{1}{3}(x-1)$

4

Notes

1. •[4] is only available if a midpoint **and** a perpendicular gradient are used.
2. Candidates who use $y = mx + c$ must obtain a numerical value for c before •[4] is available.

Regularly occurring responses

Response 1 : Candidates who use wrong midpoint or no midpoint

Candidate A

midpoint M(2, −6) ✗	✗	•[1]
$m_{MQ} = -5$ ✓̸	✓̸	•[2]
$m_{\perp} = \frac{1}{5}$ ✓̸	✓̸	•[3]
$y-(-6)=\frac{1}{5}(x-2)$ ✓̸	✓̸	•[4]

Candidate B

$m_{PQ} = -3$ ✓	✗	•[1]
$m_{\perp} = \frac{1}{3}$ ✓	✓	•[2]
^using R, $y-(-2)=\frac{1}{3}(x-1)$ ✗	✓	•[3]
	✗	•[4]

23 (b) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, −2).	2

Generic Scheme | **Illustrative Scheme**

23 (b)

•[5]	ic	use parallel gradients	•[5]	−3 **stated, or implied by** •[6]
•[6]	ic	state equation of line	•[6]	$y-(-2)=-3(x-1)$

2

Notes

3. •[6] is only available to candidates who use R and their gradient of PQ from (a).

Regularly occurring responses

Response 2 : Not using parallel gradient for equation

Candidate C

$y-(-2)=\frac{1}{3}(x-1)$ ✗

•[5] ✗
•[6] ✗

Candidate D

Parallel so same gradients

so $m=\frac{1}{3}$ ✗

$y-(-2)=\frac{1}{3}(x-1)$

•[5] ✗
•[6] ✓̸

Candidate E

$m=-3$ ✓

$y-(-2)=\frac{1}{3}(x-1)$ ✗

•[5] ✓
•[6] ✗

If $m_{PQ} = -3$ only do not award •[5]

23 (c)	Find the point of intersection of ℓ_1 and ℓ_2.	3

Generic Scheme | **Illustrative Scheme**

23 (c)

•[7]	ss	use valid approach	•[7] e.g. $x-3y=-8$ **and** $9x+3y=3$ **or** $-3x+1=\frac{1}{3}x+\frac{8}{3}$ **or** $3(3y-8)+y=1$
•[8]	pd	solve for one variable	•[8] e.g. $x=-\frac{1}{2}$
•[9]	pd	solve for other variable	•[9] e.g. $y=\frac{5}{2}$

3

Notes

4. **Neither** $x-3y=-8$ and $3x+y=1$ **nor** $y=-3x+1$ and $3y=x+8$ are sufficient to gain •[7].

5. •[7], •[8] and •[9] are not available to candidates who:
   - Equate zeros
   - Give answers only, without working
   - Use R for equations in both (a) and (b)
   - Use the same gradient for the lines in (a) and (b).

23 (d)	Hence find the shortest distance between PQ and ℓ_2.	2

Generic Scheme | **Illustrative Scheme**

23 (d)

•[10]	ss	identify appropriate points	•[10] (1, 3) and $\left(-\frac{1}{2},\frac{5}{2}\right)$
•[11]	pd	calculate distance	•[11] $\sqrt{\frac{5}{2}}$ accept $\frac{\sqrt{10}}{2}$ or $\sqrt{2\cdot5}$

2

Notes

6. •[10] and •[11] are only available for considering the distance between the midpoint of PQ and the candidate's answer from (c) **or** for considering the perpendicular distance from P or Q to ℓ_2.

7. At least one coordinate at •[10] stage must be a fraction for •[11] to be available.

8. There should only be one calculation of a distance to gain •[11].

Regularly occurring responses

Response 3 : Following through from correct (a), (b) and (c)

Candidate F

$(1, 3), (1, -2)$ ✗ •[10]

$d = 5$ ✗ •[11]

Response 4 : Following through from correct (a), (b) and (c)

Candidate G

$(1, 3), \left(-\frac{1}{2},\frac{5}{2}\right)$ ✓ •[10]

$PR=\sqrt{5}$, $QR=\sqrt{125}$, $d=\sqrt{2\cdot5}$

so $\sqrt{2\cdot5}$ is shortest distance. ✗ •[11]

If reference was made to this being the perpendicular distance then •[11] would be available.