

X100/12/03

NATIONAL
QUALIFICATIONS 2012

MONDAY, 21 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(a) Find expressions for:

- (i) $f(g(x))$;
- (ii) $g(f(x))$.

3

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.

3

2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

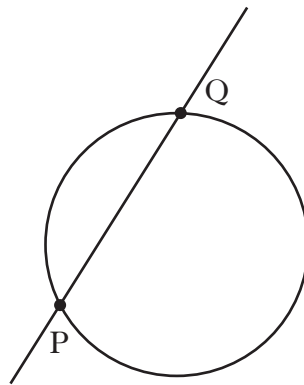


Diagram 1

Find the coordinates of P and Q.

6

(b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

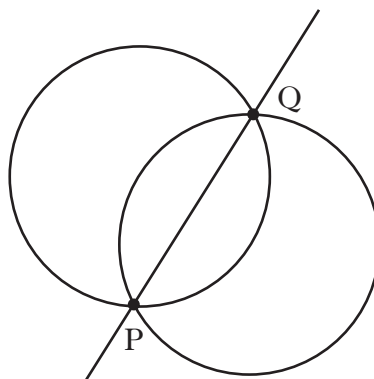


Diagram 2

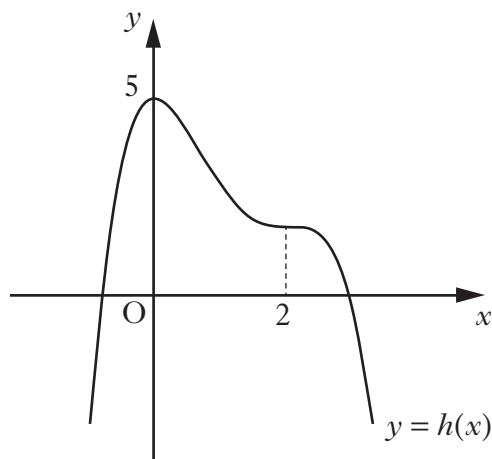
Determine the equation of this second circle.

6

3. A function f is defined on the domain $0 \leq x \leq 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.
Determine the maximum and minimum values of f .

7

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

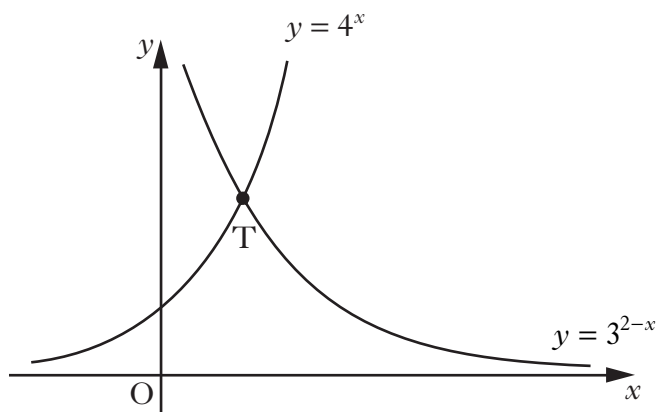
- (a) $y = h'(x)$; 3
- (b) $y = 2 - h'(x)$. 3
5. A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.
- (a) (i) Express \vec{BA} and \vec{BC} in component form. 7
- (ii) Show that $\cos \hat{ABC} = \frac{3}{\sqrt{2(k^2 + 6k + 14)}}$. 7
- (b) If angle $ABC = 30^\circ$, find the possible values of k . 5

6. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

- (a) Why do these sequences have a limit? 2
- (b) The limit of one sequence generated by this recurrence relation is $\frac{1}{2} \sin x$.
Find the value(s) of x . 7

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

- (a) Show that the x -coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$,
for all $a > 1$. 6
- (b) Calculate the y -coordinate of T. 2

[END OF QUESTION PAPER]