National
Qualifications
2017

2017 Mathematics Paper 1 (Non-calculator)

Higher

Finalised Marking Instructions

Specific marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] evaluate expression	•[1] 10	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		•[2] interpret notation •[3] state expression for $g\left(f\left(x\right)\right)$	•[2] $g(5x)$ •[3] $2\cos 5x$	2

Notes:

1. For $2\cos 5x$ without working, award both •[2] and •[3].
2. Candidates who interpret the composite function as either $g(x)\times f(x)$ or $g(x)+f(x)$ do not gain any marks.
3. $g\left(f\left(x\right)\right)=10\cos x$ award •[2]. However, $10\cos x$ with no working does not gain any marks.
4. $g\left(f\left(x\right)\right)$ leading to $2\cos(5x)$ followed by incorrect 'simplification' of the function award •[2] and •[3].

Commonly Observed Responses:

Candidate A $g\left(f\left(x\right)\right)=2\cos\left(5x\right)$ •[2]✓ •[3]✓ $=10\cos\left(x\right)$	

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ state coordinates of centre •² find gradient of radius •³ state perpendicular gradient •⁴ determine equation of tangent	•¹ (4, 3) •² $\frac{1}{3}$ •³ -3 •⁴ $y=-3x-5$	4

Notes:

1. Accept $\frac{2}{6}$ for •².
2. The perpendicular gradient must be simplified at •³ or •⁴ stage for •³ to be available.
3. •⁴ is only available as a consequence of trying to find and use a perpendicular gradient.
4. At •⁴, accept $y+3x+5=0$, $y+3x=-5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ start to differentiate •² complete differentiation	•¹ $12(4x-1)^{11}$..... •² $.......\times 4$	2

Notes:

1. •² is awarded for correct application of the chain rule.

Commonly Observed Responses:

Candidate A	Candidate B
$\frac{dy}{dx}=12(4x-1)^{11}\times 4$ •¹✓ •²✓ $\frac{dy}{dx}=36(4x-1)^{11}$ **Working subsequent to a correct answer: General Marking Principle (n)**	$\frac{dy}{dx}=36(4x-1)^{11}$ •¹✗ •²✗ **Incorrect answer with no working**

Question			Generic scheme	Illustrative scheme	Max mark
4.			Method 1 •1 use the discriminant •2 apply condition and simplify •3 determine the value of k	Method 1 •1 $4^2 - 4\times1\times(k-5)$ •2 $36-4k=0$ or $36=4k$ •3 $k=9$	3
			Method 2 •1 communicate and express in factorised form •2 expand and compare •3 determine the value of k	Method 2 •1 equal roots $\Rightarrow x^2+4x+(k-5)=(x+2)^2$ •2 x^2+4x+4 leading to $k-5=4$ •3 $k=9$	

Notes:

1. At the •1 stage, treat $4^2-4\times1\times k-5$ as bad form only if the candidate treats '$k-5$' as if it is bracketed in their next line of working. See Candidates A and B.
2. In Method 1 if candidates use any condition other than 'discriminant $=0$' then •2 is lost and •3 is unavailable.

Commonly Observed Responses:

Candidate A		Candidate B		
$4^2-4\times1\times k-5$	•1 ✓	$4^2-4\times1\times k-5$	•1 ✗	
$36-4k=0$	•2 ✓	$11-4k=0$	•2 ✓1	
$k=9$	•3 ✓	$k=\frac{11}{4}$	•3 ✓1	

Question		Generic scheme	Illustrative scheme	Max mark
5.	(a)	•[1] evaluate scalar product	•[1] 1	**1**

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
5.	(b)	•[2] calculate $\lvert\mathbf{u}\rvert$	•[2] $\sqrt{27}$	
		•[3] use scalar product	•[3] $\sqrt{27}\times\sqrt{3}\times\cos\frac{\pi}{3}$	
		•[4] evaluate $\mathbf{u.w}$	•[4] $\frac{9}{2}$ or $4{\cdot}5$	**3**

Notes:

1. Candidates who treat negative signs with a lack of rigour and arrive at $\sqrt{27}$ gain •[2].
2. Surds must be fully simplified for •[4] to be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1** ●¹ equate composite function to x ●² write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$ ●³ state inverse function	**Method 1** ●¹ $h\left(h^{-1}(x)\right)=x$ ●² $\left(h^{-1}(x)\right)^3+7=x$ ●³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3
			Method 2 ●¹ write as $y=x^3+7$ and start to rearrange ●² complete rearrangement ●³ state inverse function	**Method 2** ●¹ $y-7=x^3$ ●² $x=\sqrt[3]{y-7}$ ●³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3
			Method 3 ●¹ interchange variables ●² complete rearrangement ●³ state inverse function	**Method 3** ●¹ $x=y^3+7$ ●² $y=\sqrt[3]{x-7}$ ●³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	3

Notes:

1. $y=\sqrt[3]{x-7}$ $\left(\text{or } y=(x-7)^{\frac{1}{3}}\right)$ does not gain ●³.

2. At ●³ stage, accept h^{-1} expressed in terms of any dummy variable eg $h^{-1}(y)=\sqrt[3]{y-7}$.

3. $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$ with no working gains 3/3.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate A

$x \rightarrow x^3 \rightarrow x^3 + 7 = h(x)$ $\wedge 3 \rightarrow +7$ $\therefore -7 \rightarrow \sqrt[3]{\ }$	•1 ✓ awarded for knowing to perform the inverse operations in reverse order
$\sqrt[3]{x-7}$	•2 ✓
$h^{-1}(x) = \sqrt[3]{x-7}$	•3 ✓

Candidate B - BEWARE	**Candidate C**
$h'(x) = \ldots$ •3 ✗	$h^{-1}(x) = \sqrt[3]{x} - 7$ •3 ✗ **With no working 0/3**

Question			Generic scheme	Illustrative scheme	Max mark
7.			$\bullet^1$ **find midpoint of AB**	$\bullet^1$ $(2,7)$	
			$\bullet^2$ demonstrate the line is vertical	$\bullet^2$ m_{median} undefined	
			$\bullet^3$ state equation	$\bullet^3$ $x=2$	**3**

Notes:

1. $m_{median} = \frac{\pm 4}{0}$ alone is not sufficient to gain $\bullet^2$. Candidates must use either ‘vertical’ or ‘undefined’. However $\bullet^3$ is still available.
2. ‘$m_{median} = \frac{4}{0}$ ✗’ ‘$m_{median} = \frac{4}{0}$ impossible’ ‘$m_{median} = \frac{4}{0}$ infinite’ are **not** acceptable for $\bullet^2$. However, if these are followed by either ‘vertical’ or ‘undefined’ then award $\bullet^2$, and $\bullet^3$ is still available.
3. ‘$m_{median} = \frac{4}{0} = 0$ undefined’ ‘$m_{median} = \frac{}{0}$ undefined’ are **not** acceptable for $\bullet^2$.
4. $\bullet^3$ is not available as a consequence of using a numeric gradient; however, see notes 5 and 6.
5. For candidates who find an incorrect midpoint (a,b), using the coordinates of A and B and find the ‘median’ through C without any further errors award 1/3. However, if $a=2$, then both $\bullet^2$ and $\bullet^3$ are available.
6. For candidates who find $15y = 2x + 121$ (median through B) or $3y = 2x + 21$ (median through A) award 1/3.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$(2,7)$ $\bullet^1$ ✓ $m = \frac{4}{0}$ $= 0$ **undefined** $\bullet^2$ ✗ $x = 2$ $\bullet^3$ ✓1	$(2,7)$ $\bullet^1$ ✓ $m = \frac{4}{0}$ $= 0$ $\bullet^2$ ✗ $y = 7$ $\bullet^3$ ✓2	$(2,7)$ $\bullet^1$ ✓ $m = \frac{4}{0}$ $\bullet^2$ ^ $y - 7 = \frac{4}{0}(x-2)$ $0 = 4x - 8$ $x = 2$ $\bullet^3$ ✗
Candidate D $(2,7)$ $\bullet^1$ ✓ Median passes through $(2,7)$ and $(2,11)$ $\bullet^2$ ✗ $x = 2$ $\bullet^3$ ✓1	**Candidate E** $(2,7)$ $\bullet^1$ ✓ Both coordinates have an x value $2 \Rightarrow$ vertical line $\bullet^2$ ✓ $x = 2$ $\bullet^3$ ✓	

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 write in differentiable form	•1 $\frac{1}{2}t^{-1}$	
			•2 differentiate	•2 $-\frac{1}{2}t^{-2}$	
			•3 evaluate derivative	•3 $-\frac{1}{50}$	3

Notes:

1. Candidates who arrive at an expression containing more than one term at •1 award 0/3.
2. •2 is only available for differentiating a term containing a negative power of t.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$2t^{-1}$	•1 ✗	$2t^{-1}$	•1 ✗	$-\frac{1}{2}t^{-2}$	•1 ✓ implied by •2 ✓
$-2t^{-2}$	•2 ✓1	$-2t^{-2}$	•2 ✓1		
$-\frac{2}{25}$	•3 ✓1	$-\frac{1}{50}$	•3 ✗	$-\frac{1}{50}$	•3 ✓

Candidate D		Candidate E		Candidate F Bad form of chain rule		Candidate G	
$(2t)^{-1}$	•1 ✓	$(2t)^{-1}$	•1 ✓	$2t^{-1}$	•1 ✓	$2t^{-1}$	•1 ✗
$-(2t)^{-2}$	•2 ✗	$-(2t)^{-2}$	•2 ✗	$-2t^{-2}\times 2$	•2 ✓	$-2t^{-2}\times 2$	•2 ✗
$-\frac{1}{100}$	•3 ✓1	$-\frac{2}{25}$	•3 ✗	$-\frac{1}{50}$	•3 ✓	$-\frac{4}{25}$	•3 ✓1

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] interpret information •[2] state the value of m	•[1] $13 = 28m + 6$ **stated explicitly or in a rearranged form** •[2] $m = \frac{1}{4}$ or $m = 0 \cdot 25$	**2**

Notes:

1. Stating '$m = \frac{1}{4}$' or simply writing '$\frac{1}{4}$' with no other working gains only •[2].

Commonly Observed Responses:

Candidate A		**Candidate B**	
$13 = 28u_n + 6$	•[1] ✗	$28 = 13m + 6$	•[1] ✗
$u_n = \frac{1}{4}$	•[2] ✓1	$m = \frac{22}{13}$	•[2] ✓1

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(i)	•³ communicate condition for limit to exist	•³ a limit exists as the recurrence relation is linear and $-1<\frac{1}{4}<1$	**1**

Notes:

2. For •³ accept:

   any of $-1<\frac{1}{4}<1$ or $\left|\frac{1}{4}\right|<1$ or $0<\frac{1}{4}<1$ with no further comment;

   or statements such as:

   "$\frac{1}{4}$ lies between -1 and 1" or "$\frac{1}{4}$ is a proper fraction"

3. •³ is not available for:

   $-1\leq\frac{1}{4}\leq1$ or $\frac{1}{4}<1$

   or statements such as:

   "It is between -1 and 1." or "$\frac{1}{4}$ is a fraction."

4. Candidates who state $-1<m<1$ can only gain •³ if it is **explicitly stated** that $m=\frac{1}{4}$ in part (a).

5. Do not accept '$-1<a<1$' for •³.

Commonly Observed Responses:

Candidate C			Candidate D		
(a)	$m=\frac{1}{4}$	•¹ ✓ •² ✓	(a)	$\frac{1}{4}$	•¹ ✓ •² ✓
(b)	$-1<m<1$	•³ ✓	(b)	$-1<m<1$	•³ ✗

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(ii)	•4 know how to calculate limit •5 calculate limit	•4 $\frac{6}{1-\frac{1}{4}}$ or $L=\frac{1}{4}L+6$ •5 8	2

Notes:

6. Do not accept $L=\frac{b}{1-a}$ with no further working for •4.
7. •4 and •5 are not available to candidates who conjecture that $L=8$ following the calculation of further terms in the sequence.
8. For $L=8$ with no working, award 0/2.
9. For candidates who use a value of m appearing ex nihilo or which is inconsistent with their answer in part (a) •4 and •5 are not available.

Commonly Observed Responses:

Candidate E - no valid limit **(a)** $m=4$ •1 ✗ **(b)** $L=\frac{6}{1-4}$ •4 ✓1 $L=-2$ •5 ✗	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)			**Method 1**	
			•¹ know to integrate between appropriate limits	•¹ $\int_0^2 \dots\, dx$	
			•² use "*upper* – *lower*"	•² $\int_0^2 \left(\left(x^3-4x^2+3x+1\right)-\left(x^2-3x+1\right)\right)$	
			•³ integrate	•³ $\frac{x^4}{4}-\frac{5x^3}{3}+3x^2$	
			•⁴ substitute limits	•⁴ $\left(\frac{2^4}{4}-\frac{5\times2^3}{3}+3\times2^2\right)-(0)$	
			•⁵ evaluate area	•⁵ $\frac{8}{3}$	
				Method 2	
			•¹ know to integrate between appropriate limits for both integrals	•¹ $\int_0^2 \dots\, dx$ and $\int_0^2 \dots\, dx$	
			•² integrate both functions	•² $\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}+x$ and $\frac{x^3}{3}-\frac{3x^2}{2}+x$	
			•³ substitute limits into both functions	•³ $\left(\frac{2^4}{4}-\frac{4\left(2^3\right)}{3}+\frac{3\left(2^2\right)}{2}+2\right)-0$ and $\left(\frac{2^3}{3}-\frac{3\left(2^2\right)}{2}+2\right)-0$	
			•⁴ evaluation of both functions	•⁴ $\frac{4}{3}$ and $\frac{-4}{3}$	
			•⁵ evidence of subtracting areas	•⁵ $\frac{4}{3}-\frac{-4}{3}=\frac{8}{3}$	**5**

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. $\bullet^1$ is not available to candidates who omit 'dx'.
2. Treat the absence of brackets at $\bullet^2$ stage as bad form only if the correct integral is obtained at $\bullet^3$. See Candidates A and B.
3. Where a candidate differentiates one or more terms at $\bullet^3$, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are unavailable.
4. Accept unsimplified expressions at $\bullet^3$ e.g. $\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}+x-\frac{x^3}{3}+\frac{3x^2}{2}-x$.
5. Do not penalise the inclusion of '$+c$'.
6. Candidates who substitute limits without integrating do not gain $\bullet^3$, $\bullet^4$ or $\bullet^5$.
7. $\bullet^4$ is only available if there is evidence that the lower limit '0' has been considered.
8. Do not penalise errors in substitution of $x=0$ at $\bullet^3$.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ✓

$\int_0^2 x^3-4x^2+3x+1-x^2-3x+1\ dx$

$\frac{x^4}{4}-\frac{5x^3}{3}+3x^2$ $\bullet^3$ ✓ $\Rightarrow \bullet^2$ ✓

Candidate B

$\bullet^1$ ✓

$\int_0^2 x^3-4x^2+3x+1-x^2-3x+1\ dx$ $\bullet^2$ ✗

$\frac{x^4}{4}-\frac{5x^3}{3}+2x$ $\bullet^3$ ✓1

$\int \ldots = -\frac{16}{3}$ cannot be negative so $=\frac{16}{3}$ $\bullet^5$ ✗

However, $\int \ldots = -\frac{16}{3}$ so Area $=\frac{16}{3}$ $\bullet^5$ ✓

Treating individual integrals as areas

Candidate C - Method 2

$\bullet^1$ ✓

$\bullet^2$ ✓

$\bullet^3$ ✓

$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$ ✓

$\therefore$ Area is $\frac{4}{3}-\left(-\frac{4}{3}\right)=\frac{8}{3}$ $\bullet^5$ ✓

Candidate D - Method 2

$\bullet^1$ ✓

$\bullet^2$ ✓

$\bullet^3$ ✓

$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$ ✓

$=\frac{4}{3}$

$\therefore$ Area is $\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$ $\bullet^5$ ✗

Candidate E - Method 2

$\bullet^1$ ✓

$\bullet^2$ ✓

$\bullet^3$ ✓

$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$ ✓

Area cannot be negative

$\therefore$ Area is $\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$ $\bullet^5$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)			**Method 1**	
			•[6] use "*line – quadratic*"	•[6] $\int\left(\left(1-x\right)-\left(x^2-3x+1\right)\right)dx$	
			•[7] integrate	•[7] $-\frac{x^3}{3}+x^2$	
			•[8] substitute limits and evaluate integral	•[8] $\left(-\frac{2^3}{3}+2^2\right)-\left(0\right)=\frac{4}{3}$	
			•[9] state fraction	•[9] $\frac{1}{2}$	
				Method 2	
			•[6] use "cubic - *line*"	•[6] $\int\left((x^3-4x^2+3x+1)-\left(1-x\right)\right)dx$	
			•[7] integrate	•[7] $\frac{x^4}{4}-\frac{4x^3}{3}+2x^2$	
			•[8] substitute limits and evaluate integral	•[8] $\left(\frac{2^4}{4}-4\times\frac{2^3}{3}+2\times2^2\right)-\left(0\right)=\frac{4}{3}$	
			•[9] state fraction	•[9] $\frac{1}{2}$	
				Method 3	
			•[6] integrate line	•[6] $\int\left(1-x\right)dx=\left[x-\frac{x^2}{2}\right]_0^2$	
			•[7] substitute limits and evaluate integral	•[7] $\left(2-\frac{2^2}{2}\right)-\left(0\right)=0$	
			•[8] evidence of subtracting integrals	•[8] $0-\left(-\frac{4}{3}\right)=\frac{4}{3}$ or $\frac{4}{3}-0$	
			•[9] state fraction	•[9] $\frac{1}{2}$	4

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

IMPORTANT: Notes prefixed by * may be subject to General Marking Principle (n). If a candidate has been penalised for the error in (a) then they must not be penalised a second time for the same error in (b).**

9. *** $\bullet^6$ is not available to candidates who omit 'dx'.
10. In Methods 1 and 2 only, treat the absence of brackets at $\bullet^6$ stage as bad form only if the correct integral is obtained at $\bullet^7$.
11. Candidates who have an incorrect expression to integrate at the $\bullet^3$ and $\bullet^7$ stage due solely to the absence of brackets lose $\bullet^2$, but are awarded $\bullet^6$.
12. Where a candidate differentiates one or more terms at $\bullet^7$, then $\bullet^7$, $\bullet^8$ and $\bullet^9$ are unavailable.
    *** In cases where Note 3 has applied in part (a), $\bullet^7$ is lost but $\bullet^8$ and $\bullet^9$ are available.
13. In Methods 1 and 2 only, accept unsimplified expressions at $\bullet^7$ e.g. $x-\frac{x^2}{2}-\frac{x^3}{3}+\frac{3x^2}{2}-x$
14. Do not penalise the inclusion of '$+c$'.
15. *** $\bullet^8$ in Methods 1 and 2 and $\bullet^7$ in method 3 is only available if there is evidence that the lower limit '0' has been considered.
16. At the $\bullet^9$ stage, the fraction must be consistent with the answers at $\bullet^5$ and $\bullet^8$ for $\bullet^9$ to be awarded.
17. Do not penalise errors in substitution of $x=0$ at $\bullet^8$ in Method 1 & 2 or $\bullet^7$ in Method 3.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.				**Method 1**	
			•[1] determine the gradient of given line or of AB	•[1] $\frac{2}{3}$ or $\frac{a-2}{12}$	
			•[2] determine the other gradient	•[2] $\frac{a-2}{12}$ or $\frac{2}{3}$	
			•[3] find a	•[3] 10	
				Method 2	
			•[1] determine the gradient of given line	•[1] $\frac{2}{3}$ stated or implied by •[2]	
			•[2] equation of line and substitute	•[2] $y-2=\frac{2}{3}(x+7)$ $a-2=\frac{2}{3}(5+7)$	
			•[3] solve for a	•[3] 10	3

Notes:

Commonly Observed Responses:

Candidate A - using simultaneous equations	Candidate B	Candidate C - Method 2
$m_{\text{line}}=\frac{2}{3}$ •[1] ✓ $3y=2x+20$ $3y=2x-10+3a$ •[2] ✓ $0=0+30-3a$ $3a=30$ $a=10$ •[3] ✓	$m_{AB}=\frac{a-2}{12}$ •[1] ✓ $\frac{a-2}{12}=-2$ •[2] ✗ $a=-22$ •[3] ✓1	•[1] ✓ $y-2=\frac{2}{3}(x+7)$ $3y=2x+20$ $3y=2\times5+20$ •[2] ✓ $3y=30$ $y=10$ No mention of a •[3] ^

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ use laws of logs •² write in exponential form •³ solve for a	•¹ $\log_a 9$ •² $a^{\frac{1}{2}} = 9$ •³ 81	3

Notes:

1. $\frac{36}{4}$ must be simplified at •¹ or •² stage for •¹ to be awarded.
2. Accept $\log 9$ at •¹.
3. •² may be implied by •³.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$\log_a 144$ •¹ ✗ $a^{\frac{1}{2}} = 144$ •² ✓1 $a = 12$ •³ ✗	$\log_a 32$ •¹ ✗ $a^{\frac{1}{2}} = 32$ •² ✓1 •³ ^	$\log_a 9$ •¹ ✓ $a = 9^{\frac{1}{2}}$ •² ✗ $a = 3$ •³ ✓2
Candidate D $2\log_a 36 - 2\log_a 4 = 1$ $\log_a 36^2 - \log_a 4^2 = 1$ •¹ ✓ $\log_a \frac{36^2}{4^2} = 1$ $\log_a 81 = 1$ •² ✓ $a = 81$ •³ ✓		

Question			Generic scheme	Illustrative scheme	Max mark
13.			•¹ write in integrable form •² start to integrate •³ process coefficient of x •⁴ complete integration and simplify	•¹ $(5-4x)^{-\frac{1}{2}}$ •² $\frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$ •³ $\ldots\times\frac{1}{(-4)}$ •⁴ $-\frac{1}{2}(5-4x)^{\frac{1}{2}}+c$	4

Notes:

1. For candidates who differentiate throughout, only •¹ is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/4.
3. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket no further marks are available.
4. '$+c$' is required for•⁴.

Commonly Observed Responses:

Candidate A		Candidate B	
$(5-4x)^{-\frac{1}{2}}$	•¹ ✓	$(5-4x)^{\frac{1}{2}}$	•¹ ✗
$\frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}$	•² ✓ •³ ^	$\frac{(5-4x)^{\frac{3}{2}}}{\frac{3}{2}}\times\frac{1}{(-4)}$	•² ✓1 •³ ✓
$2(5-4x)^{\frac{1}{2}}+c$	•⁴ ✓2	$-\frac{(5-4x)^{\frac{3}{2}}}{6}+c$	•⁴ ✓1

Candidate C		Candidate D	
Differentiate in part:		Differentiate in part:	
$(5-4x)^{-\frac{1}{2}}$	•¹ ✓	$(5-4x)^{-\frac{1}{2}}$	•¹ ✓
$-\frac{1}{2}(5-4x)^{-\frac{3}{2}}\times\frac{1}{(-4)}$	•² ✗ •³ ✓	$\frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\times(-4)$	•² ✓ •³ ✗
$\frac{1}{8}(5-4x)^{-\frac{3}{2}}+c$	•⁴ ✓1	$-8(5-4x)^{\frac{1}{2}}+c$	•⁴ ✓1

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		•[1] use compound angle formula •[2] compare coefficients •[3] process for k •[4] process for a and express in required form	•[1] $k\sin x°\cos a° - k\cos x°\sin a°$ **stated explicitly** •[2] $k\cos a° = \sqrt{3},\ k\sin a° = 1$ **stated explicitly** •[3] $k = 2$ •[4] $2\sin(x-30)°$	4

Notes:

1. Accept $k(\sin x°\cos a° - \cos x°\sin a°)$ for •[1]. Treat $k\sin x°\cos a° - \cos x°\sin a°$ as bad form only if the equations at the •[2] stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\sin x°\cos a° - 2\cos x°\sin a°$ or $2(\sin x°\cos a° - \cos x°\sin a°)$ is acceptable for •[1] and •[3].
4. In the calculation of $k = 2$, do not penalise the appearance of -1.
5. Accept $k\cos a° = \sqrt{3},\ -k\sin a° = -1$ for •[2].
6. •[2] is not available for $k\cos x° = \sqrt{3},\ k\sin x° = 1$, however, •[4] is still available.
7. •[3] is only available for a single value of $k,\ k > 0$.
8. •[3] is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without simplifying at any stage.
9. •[4] is not available for a value of a given in radians.
10. Candidates may use any form of the wave equation for •[1], •[2] and •[3], however, •[4] is only available if the value of a is interpreted in the form $k\sin(x-a)°$
11. Evidence for •[4] may only appear as a label on the graph in part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	Candidate B	
•[1] ^ $2\cos a = \sqrt{3}$ $2\sin a = 1$ •[2] ✓ •[3] ✓ $\tan a = \frac{1}{\sqrt{3}},\ a = 30$ $2\sin(x-30)°$ •[4] ✓	$k\sin x\cos a - k\cos x\sin a$ •[1] ✓ $\cos a = \sqrt{3}$ $\sin a = 1$ •[2] ✗ $\tan a = \frac{1}{\sqrt{3}}$ $a = 30$ (Not consistent with equations at •[2].) $2\sin(x-30)°$ •[3] ✓ •[4] ✗	

<table>
<tr><th>Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td colspan="4">Responses with the correct expansion of $k\sin(x-a)°$ but errors for either $•^2$ or $•^4$.</td></tr>
<tr><td colspan="4">
<table>
<tr>
<td>Candidate C

$k\cos a=\sqrt{3},\ k\sin a=1$ $•^2$ ✓

$\tan a=\sqrt{3}$ $•^4$ ✗
$a=60$</td>
<td>Candidate D

$k\cos a=1,\ k\sin a=\sqrt{3}$ $•^2$ ✗

$\tan a=\sqrt{3}$
$a=60$
$2\sin(x-60)°$ $•^4$ ✓1</td>
<td>Candidate E

$k\cos a=\sqrt{3},\ k\sin a=-1$ $•^2$ ✗

$\tan a=-\frac{1}{\sqrt{3}},\ a=330$

$2\sin(x-330)°$ $•^4$ ✓1</td>
</tr>
</table>
</td></tr>
<tr><td colspan="4">Responses with the incorrect labelling; $k\sin A\cos B-k\cos A\sin B$ from formula list.</td></tr>
<tr><td colspan="4">
<table>
<tr>
<td>Candidate F
$k\sin A\cos B-k\cos A\sin B$ $•^1$ ✗
$k\cos a=\sqrt{3}$
$k\sin a=1$ $•^2$ ✓
$\tan a=\frac{1}{\sqrt{3}},\ a=30$
$2\sin(x-30)°$ $•^3$ ✓ $•^4$ ✓</td>
<td>Candidate G
$k\sin A\cos B-k\cos A\sin B$ $•^1$ ✗
$k\cos x=\sqrt{3}$
$k\sin x=1$ $•^2$ ✗
$\tan x=\frac{1}{\sqrt{3}},\ x=30$
$2\sin(x-30)°$ $•^3$ ✓ $•^4$ ✓1</td>
<td>Candidate H
$k\sin A\cos B-k\cos A\sin B$ $•^1$ ✗
$k\cos B=\sqrt{3}$
$k\sin B=1$ $•^2$ ✗
$\tan B=\frac{1}{\sqrt{3}},\ B=30$
$2\sin(x-30)°$ $•^3$ ✓ $•^4$ ✓1</td>
</tr>
</table>
</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		$\bullet^5$ roots identifiable from graph $\bullet^6$ coordinates of both turning points identifiable from graph $\bullet^7$ y-intercept and value of y at $x=360$ identifiable from graph	$\bullet^5$ 30 and 210 $\bullet^6$ $(120, 2)$ and $(300, -2)$ $\bullet^7$ -1	3

Notes:

12. $\bullet^5$, $\bullet^6$and $\bullet^7$ are only available for attempting to draw a "sine" graph with a period of 360°.
13. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
14. Vertical marking is not applicable to $\bullet^5$ and $\bullet^6$.
15. Candidates sketch arrived at in (b) must be consistent with the equation obtained in (a), see also candidates I and J.
16. For any incorrect horizontal translation of the graph of the wave function arrived at in part(a) only $\bullet^6$ is available.

Commonly Observed Responses:

Candidate I	**Candidate J**
(a) $2\sin(x-30)$ correct equation (b) Incorrect translation: Sketch of $2\sin(x+30)$ Only $\bullet^6$ is available	(a) $2\sin(x+30)$ incorrect equation (b) Sketch of $2\sin(x+30)$ All 3 marks are available

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] state value of a •[2] state value of b	•[1] -5 •[2] 3	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative Scheme	Max Mark
15.	(b)		•[3] state value of integral	•[3] 10	1

Notes:

1. Candidates answer at (b) must be consistent with the value of b obtained in (a).
2. In parts (b) and (c), candidates who have 10 and -6 accompanied by working, the working must be checked to ensure that no errors have occurred prior to the correct answer appearing.

Commonly Observed Responses:

Candidate A

From (a)

$a=-3$ •[1] ✗

$b=5$ •[2] ✗

$\int h(x)\,dx=14$ •[3] ✓1

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		•[4] state value of derivative	•[4] -6	1

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]