

[C100/SQP248]

Higher
Mathematics

Time: 1 hour 10 minutes

NATIONAL
QUALIFICATIONS

Units 1, 2 and 3

Paper 1

(Non-calculator)

Specimen Question Paper **(Revised)**

for use in and after 2004

Read Carefully

- 1 **Calculators may NOT be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

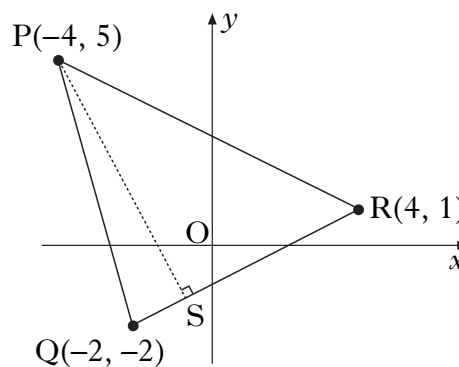
Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted.

Marks

1. $P(-4, 5)$, $Q(-2, -2)$ and $R(4, 1)$ are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P.



(4)

2. A sequence is defined by the recurrence relation $u_{n+1} = 0.3u_n + 5$ with first term u_1 .

(a) Explain why this sequence has a limit as n tends to infinity.

(1)

(b) Find the **exact** value of this limit.

(2)

3. (a) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors.

(4)

(b) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.

(2)

(c) Find the stationary points of $y = f(x)$ and determine the nature of each.

(5)

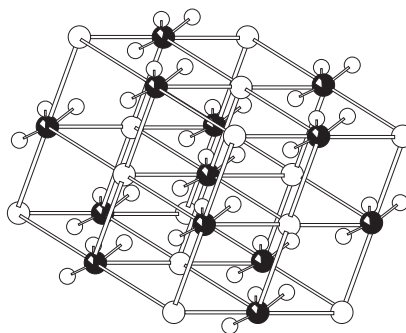
(d) Sketch the graph of $y = f(x)$.

(1)

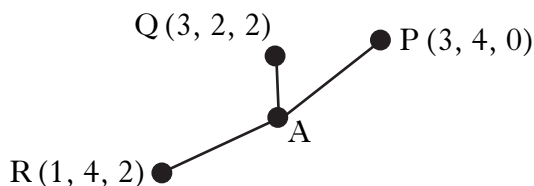
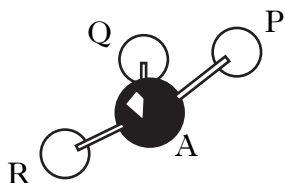
4. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x + 30)^\circ$ is $\frac{4\sqrt{3} + 3}{10}$.

(4)

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



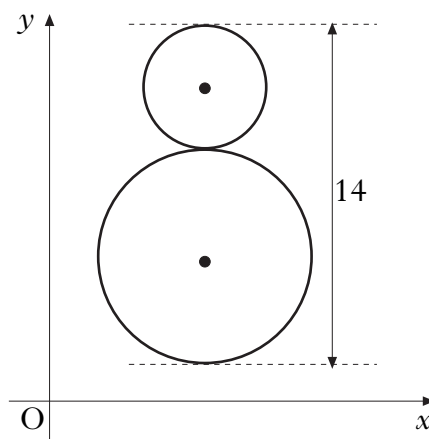
The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.



- (a) Show that the cosine of angle PQR is $\frac{1}{2}$. (7)
- (b) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.
- (i) Find the coordinates of T.
- (ii) Show that P, Q and R are equidistant from T. (7)

6. A bakery firm makes ginger-bread men each 14 cm high with a circular “head” and “body”. The equation of the “body” is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y -axis.

Find the equation of the “head”.



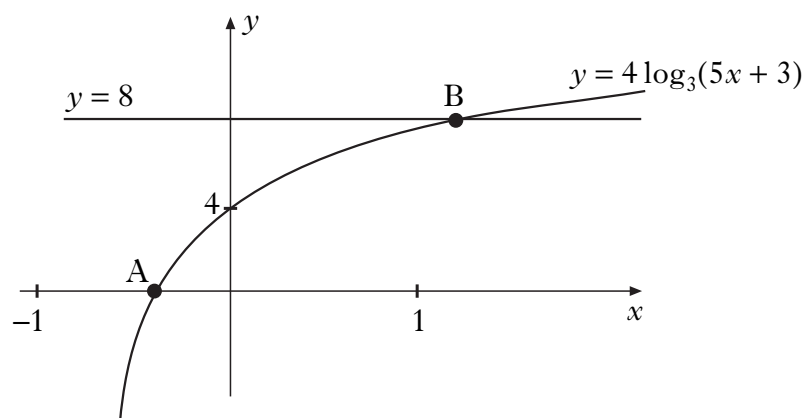
(6)

7. Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2} du$. (7)

8. Sketch the graph of $y = 2\sin(x - 30)^\circ$ for $0 \leq x < 360$. (4)

9. Find $\frac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$. (3)

10. Part of the graph of $y = 4 \log_3(5x + 3)$ is shown in the diagram. This graph crosses the x -axis at the point A and the straight line $y = 8$ at the point B. (3)
- Find the x -coordinate of B.



[END OF SPECIMEN QUESTION PAPER]