[C100/SQP248]

Higher Mathematics

NATIONAL QUALIFICATIONS

Units 1, 2 and 3
Paper 1
Specimen Marking Instructions **(Revised)**

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (st), which marks awarded are processing marks (pr) and which marks awarded are interpretation and communication marks (ic).

SCOTTISH QUALIFICATIONS AUTHORITY

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
1	**ans: $y = -2x - 3$ — 4 marks** $\bullet^{1}$ st: know to get m_{QR} then m_{PS} $\bullet^{2}$ pr: use gradient formula $\bullet^{3}$ st: know how to find perp. gradient $\bullet^{4}$ ic: state equation of st. line	 $\bullet^{1}$ strat: find m_{QR} then m_{PS} $\bullet^{2}$ $m_{QR} = \frac{1}{2}$ $\bullet^{3}$ $m_{PS} = -2$ $\bullet^{4}$ PS: $y - 5 = -2(x + 4)$
2	**ans: $-1 < 0{\cdot}3 < 1$, $\frac{50}{7}$ — 3 marks** $\bullet^{1}$ ic: state condition for limit $\bullet^{2}$ st: know how to find limit $\bullet^{3}$ pr: complete strategy for exact limit	 $\bullet^{1}$ $-1 < 0{\cdot}3 < 1$ $\bullet^{2}$ *eg* $L = 0{\cdot}3L + 5$ $\bullet^{3}$ $L = \frac{50}{7}$
3a	**ans: $f(1) = 0$, $(x - 4)$, $(x - 1)$ — 4 marks** $\bullet^{1}$ st: know to find factor of cubic $\bullet^{2}$ pr: reach zero at finish $\bullet^{3}$ ic: state quadratic factor $\bullet^{4}$ pr: complete factorisation	 $\bullet^{1}$ *eg* 1 | 1 −6 9 −4 1 $\bullet^{2}$ *eg* 1 | 1 −6 9 −4 1 −5 4 1 −5 4 0 $\bullet^{3}$ $x^2 - 5x + 4$ $\bullet^{4}$ $(x - 4)(x - 1)$
3b	**ans: (1,0), (4,0), (0,–4) — 2 marks** $\bullet^{5}$ ic: x-axis intersections $\bullet^{6}$ ic: y-axis intersections	 $\bullet^{5}$ (1,0), (4,0) $\bullet^{6}$ (0,–4)
3c	**ans: max at (1,0), min at (3,–4) — 5marks** $\bullet^{7}$ st: know to set derivative = 0 $\bullet^{8}$ pr: differentiate $\bullet^{9}$ pr: find stationary points $\bullet^{10}$ st: know how to test nature $\bullet^{11}$ ic: complete nature test	 $\bullet^{7}$ $\frac{dy}{dx} = 0$ $\bullet^{8}$ $3x^2 - 12x + 9$ $\bullet^{9}$ (1,0), (3,–4) $\bullet^{10}$ *eg* x: . . . 1 . . . 3 . . . y': + 0 − 0 + $\bullet^{11}$ max at (1,0), min at (3,–4)
3d	**ans: sketch — 1 mark** $\bullet^{12}$ ic: draw sketch	 $\bullet^{12}$ sketch

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4	**ans:** $\dfrac{4\sqrt{3}+3}{10}$ **4 marks** •[1] st: know to expand •[2] st: know to use right-angled trig. •[3] ic: recall exact values •[4] pr: substitute and complete proof	•[1] $\sin x^{\circ}\cos 30^{\circ}+\cos x^{\circ}\sin 30^{\circ}$ •[2] $\sin x^{\circ}=\frac{4}{5}$ ***and*** $\cos x^{\circ}=\frac{3}{5}$ •[3] $\sin 30^{\circ}=\frac{1}{2}$ ***and*** $\cos 30^{\circ}=\frac{\sqrt{3}}{2}$ •[4] $\dfrac{4\sqrt{3}+3}{10}$
5a	**ans: proof** **7 marks** •[1] ic: find vector components •[2] ic: find vector components •[3] st: eg know to use scalar product •[4] ic: find scalar product •[5] pr: find magnitude •[6] pr: find magnitude •[7] pr: complete proof	•[1] $\overrightarrow{PQ}=\begin{pmatrix}0\\-2\\2\end{pmatrix}$ •[2] $\overrightarrow{RQ}=\begin{pmatrix}2\\-2\\0\end{pmatrix}$ •[3] $\cos PQR=\dfrac{\overrightarrow{PQ}.\overrightarrow{RQ}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{RQ}\rvert}$ •[4] $\overrightarrow{PQ}.\overrightarrow{RQ}=4$ •[5] $PQ=\sqrt{8}$ •[6] $RQ=\sqrt{8}$ •[7] substitution leading to $\frac{1}{2}$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
5b	**ans: M(2,3,2) T$\left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$; proof** **7 marks** •[8] ic: find coordinates of M •[9] ic: interpret ratio •[10] pr: find appropriate vector •[11] pr: find appropriate vector •[12] pr: complete calc. of coordinates of T •[13] st: know how to find distance in 3D •[14] pr: complete proof	•[8] $M = (2,3,2)$ •[9] *eg* $\overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ •[10] $\overrightarrow{PM} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ •[11] $\overrightarrow{PT} = \begin{pmatrix} -\frac{2}{3} \\ -\frac{2}{3} \\ \frac{4}{3} \end{pmatrix}$ •[12] $T = \left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$ •[13] $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ •[14] $PT = 2\sqrt{\frac{2}{3}}, \quad QT = 2\sqrt{\frac{2}{3}}, \quad RT = 2\sqrt{\frac{2}{3}}$
6	**ans: $(x-5)^2 + (y-13)^3 = 9$ 6 marks** •[1] ic: state radius from equ. of circle •[2] ic: interpret diagram •[3] ic: interpret distance between centres •[4] ic: state centre from equ. of circle •[5] ic: interpret diagram •[6] ic: state equation of circle	•[1] $radius_{body} = 4$ •[2] $radius_{head} = 3$ •[3] distance between centres = 7 •[4] $centre_{body} = (5,6)$ •[5] $centre_{head} = (5,13)$ •[6] $(x-5)^2 + (y-13)^3 = 9$
7	**ans: 1** **7 marks** •[1] st: know to split into sums/differences •[2] pr: write in integrable form •[3] pr: write in integrable form •[4] pr: integrate •[5] pr: integrate •[6] pr: substitute limits of integration •[7] pr: evaluate	•[1] $\frac{u^2}{2u^2} + \frac{2}{2u^2}$ •[2] $\frac{1}{2}$ •[3] u^{-2} •[4] $\frac{1}{2}u$ •[5] $-u^{-1}$ •[6] $\left(\frac{1}{2}\times 2 - 2^{-1}\right) - \left(\frac{1}{2}\times 1 - 1^{-1}\right)$ •[7] 1

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
8	**ans: sketch** **4 marks** for $y = a\sin(x + b)$ •[1] ic: know "b" represents translation •[2] ic: know "a" represents vert. scaling •[3] st: know the order of transformations •[4] ic: complete sketch	•[1] "-30^{o}" means move $y = \sin x^{o}$: $\begin{pmatrix} +30° \\ 0 \end{pmatrix}$ •[2] "2" means stretch two-fold parallel to y-axis •[3] order is "-30^{o}" then "2" •[4] sketch evidence for •[1], •[2] and •[3] may be either stated, or implied by a correct sketch at the •[4] stage.
9	**ans: $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$** **3 marks** •[1] st: know how to deal with √ •[2] pr: start differentiation •[3] pr: apply chain rule	•[1] $(1 + \cos x)^{\frac{1}{2}}$ •[2] $\frac{1}{2}(1+\cos x)^{-\frac{1}{2}}$ •[3] $\times -\sin x$
10	**ans: $\frac{6}{5}$** **3 marks** •[1] ic: interpret diagram •[2] st: know how to solve log equation •[3] pr: complete the solving	•[1] $\log_3(5x + 3) = 2$ •[2] $5x + 3 = 3^2$ •[3] $x = \frac{6}{5}$

[*END OF SPECIMEN MARKING INSTRUCTIONS*]