Mathematics
Practice Paper A
Paper 1
Assessing Units 1, 2 & 3

NATIONAL QUALIFICATIONS

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\,,b)$ and radius r.

Scalar Product: $a\,.\,b = |a|\,|b|\cos\theta$, where θ is the angle between a and b.

or

$$a\,.\,b = a_1b_1 + a_2b_2 + a_3b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A\cos A
\end{aligned}$$

Table of standard

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Section A

1. The derivative of $\dfrac{10x^3 - 3}{5x}$ with respect to x is

A. $4x - \dfrac{3}{5}x^{-2}$

B. $\dfrac{2}{3}x^3 - \dfrac{3}{5}x$

C. $4x + \dfrac{3}{5}x^{-2}$

D. $2x^2 - 3$

2. A curve has equation $y = 2x^3 - 5x + 1$. The gradient of the tangent to this curve at the point where $x = 3$ is

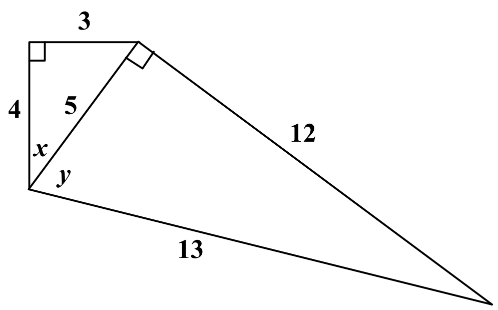
A. 50

B. 49

C. 40

D. 21

3.



The diagram shows two right-angled triangles with lengths as indicated.

The value of $\sin(x + y)$ is

A. $-\dfrac{16}{65}$

B. $\dfrac{56}{65}$

C. $-\dfrac{33}{65}$

D. $\dfrac{63}{65}$

4. A sequence is defined by the recurrence relation $U_{n+1} = aU_n - 2$ with $U_0 = 3$. An expression, in terms of a, for U_2 is

A. $6a - 4$

B. $3a^2 - 2$

C. $3a - 2$

D. $3a^2 - 2a - 2$

5. k and α are given by $k\sin\alpha = 4$ and $k\cos\alpha = -1$ where $0 \le \alpha \le \pi$. The value of k and the range of values for α are

A. $\sqrt{17}$ and $\frac{\pi}{2} < \alpha < \pi$

B. $\sqrt{17}$ and $0 < \alpha < \frac{\pi}{2}$

C. $\sqrt{15}$ and $\frac{\pi}{2} < \alpha < \pi$

D. $\sqrt{15}$ and $0 < \alpha < \frac{\pi}{2}$

6. A sequence is generated by the recurrence relation $U_{n+1} = 0 \cdot 6U_n + 7$. The limit of this sequence as $n \to \infty$ is

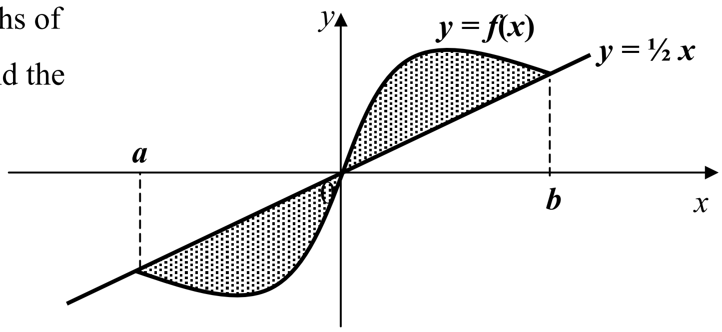
A. 7

B. $\frac{35}{2}$

C. $\frac{35}{3}$

D. $\frac{2}{5}$

7. The diagram shows part of the graphs of a function with equation $y = f(x)$ and the line with equation $y = ½\,x$.



The shaded area can be found from

A. $\int_a^0 (\frac{1}{2}x - f(x))\,dx - \int_0^b (f(x) - \frac{1}{2}x)\,dx$

B. $\int_a^b (f(x) - \frac{1}{2}x)\,dx$

C. $\int_a^b (\frac{1}{2}x - f(x))\,dx$

D. $\int_a^0 (\frac{1}{2}x - f(x))\,dx + \int_0^b (f(x) - \frac{1}{2}x)\,dx$

8. Which of the following expressions is equal to $2\sin(x + \frac{\pi}{3})$?

A. $\sqrt{3}\cos x - \sin x$

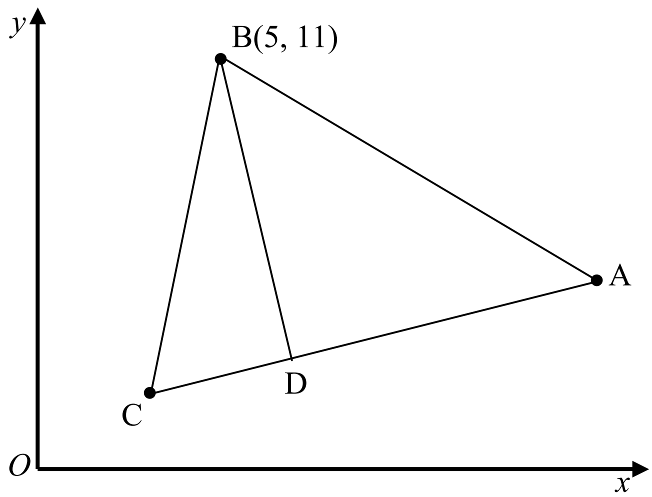
B. $\sqrt{3}\sin x - \cos x$

C. $\sin x - \sqrt{3}\cos x$

D. $\sin x + \sqrt{3}\cos x$

9. The diagram shows triangle ABC with altitude BD.

B has coordinates (5, 11) and the gradient of CA is $\frac{1}{4}$.



The equation of the altitude is

A. $4y - x - 9 = 0$

B. $y + 4x - 31 = 0$

C. $4y - x - 39 = 0$

D. $y + 4x + 9 = 0$

10. The vectors $\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ y \\ 2 \end{pmatrix}$ are perpendicular. The value of y is

A. 0

B. 4

C. -4

D. 7

11. The integral of $(3x+2)(4x-1)$ with respect to x is

A. $12x^2+5x-2+C$

B. $4x^3+\frac{5}{2}x^2-2x+C$

C. $6x^3+5x^2-2x+C$

D. $24x+5+C$

12. $\boldsymbol{a}$ and $\boldsymbol{b}$ are vectors with components $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$ respectively. If the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is x°, the value of $\cos x^\circ$ is

A. $-\frac{2}{\sqrt{6}}$

B. 0

C. $\frac{1}{12}$

D. $-\frac{1}{\sqrt{3}}$

13. A circle with centre $(-2, 7)$ passes through the point $(1, 3)$. The equation of the circle is

A. $(x+2)^2+(y-7)^2=25$

B. $(x-2)^2+(y-7)^2=5$

C. $(x+2)^2+(y-7)^2=5$

D. $(x-2)^2+(y+7)^2=25$

14. When $4x^2+16x-7$ is expressed in the form $4(x+a)^2+b$, the value of b is

A. -4

B. -15

C. -11

D. -23

15. Two functions are defined as $f(x)=\dfrac{1}{x^2}$ and $g(x)=-3x$. $f(g(x))$ equals

A. $-\dfrac{1}{9x^2}$

B. $-\dfrac{1}{3x^2}$

C. $\dfrac{1}{9x^2}$

D. $\dfrac{1}{3x^2}$

16. The point P(3, 1) lies on the circle with equation $x^2+y^2-4x-6y+8=0$. The gradient of the tangent at point P is

A. $-\dfrac{1}{2}$

B. $\dfrac{1}{2}$

C. -2

D. 2

17. $\displaystyle\int 4\sin 3x \, dx$ equals

A. $\dfrac{4}{3}\cos 3x+C$

B. $\dfrac{4}{3}\sin 3x+C$

C. $-\dfrac{4}{3}\cos 3x+C$

D. $-12\cos 3x+C$

18. Functions f and g are given by $f(x) = 2x^2 - 1$ and $g(x) = 6 - x$.

The value of $g(f(3))$ is

A. -11

B. -29

C. 17

D. 35

19. The circle with equation $(x-3)^2 + (y+2)^2 = 16$ has centre A and the circle with equation $2x^2 + 2y^2 - 4x + 12y = 0$ has centre B. The length of AB is

A. $\sqrt{17}$

B. $\sqrt{5}$

C. $\sqrt{29}$

D. $\sqrt{3}$

20. P, Q and R have coordinates $(-1,-8,-2)$, $(2,-5,4)$ and $(3,-4,6)$ respectively.

Here are two statements about the points.

(1) P, Q and R are collinear

(2) Q divides PR in the ratio 1 : 3

Which of these statements is/are true?

A. both statements

B. statement (1) only

C. statement (2) only

D. neither statement

Section B

21. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 8\, U_n + 3$.

(a) Explain why this sequence has a limit as $n \to \infty$. **(1)**

(b) Find the limit of this sequence. **(2)**

(c) Taking $U_0 = 10$ and L as the limit of the sequence, find n such that

$$L - U_n = 2 \cdot 56$$ **(3)**

22. Two circles, which do not touch or overlap, have as their equations

$$(x-15)^2 + (y-6)^2 = 40 \quad \text{and} \quad x^2 + y^2 - 6x - 4y + 3 = 0 \ .$$

(a) Show that the **exact** distance between the centres of the two circles is $4\sqrt{10}$ units. **(3)**

(b) Hence show that the shortest distance between the two circles is equal to the radius of the smaller circle. **(4)**

23. (a) Points E , F and G have coordinates (-1,2,1) , (1,3,0) and (-2,-2,2) respectively.

Given that $3\overrightarrow{EF} = \overrightarrow{GH}$, find the coordinates of the point H. **(3)**

(b) Hence calculate $\left|\overrightarrow{EH}\right|$. **(2)**

24. Solve algebraically the equation

$$3\cos 2x^\circ - 9\cos x^\circ = 12 \qquad \text{for} \qquad 0 \leq x < 360.$$ **(6)**

25. (a) Given that $2\log_x y = \log_x 2y + 2$ find a relationship connecting x and y . **(4)**

(b) Hence find y when $x = \frac{1}{4}y$ and $y > 0$. **(2)**

END OF QUESTION PAPER