NATIONAL QUALIFICATIONS

# Mathematics
# Practice Paper B
# Paper 1
# Assessing Units 1, 2 & 3

**Time allowed - 1 hour 30 minutes**

**Read carefully**

**Calculators may NOT be used in this paper.**

**Section A - Questions 1 - 20 (40 marks)**

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

**Section B (30 marks)**

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

**FORMULAE LIST**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\,, b)$ and radius $r$.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where $\theta$ is the angle between $a$ and $b$.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2 \sin A \cos A
\end{aligned}$$

Table of standard derivatives:

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

Table of standard integrals:

| $f(x)$ | $\int f(x)\, dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a} \cos ax + C$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + C$ |

**All questions should be answered**

**Section A**

**1.** The tangent to the curve $y = x^3 - 4$ is drawn at the point where $x = -2$. The gradient of this tangent is

**A.** 12

**B.** 36

**C.** $-12$

**D.** 0

**2.** The points A$(2, -1, 5)$, B$(-1, 2, -1)$ and C$(x, 4, -5)$ are collinear. The value of $x$ is

**A.** $-2$

**B.** $-1$

**C.** $-3$

**D.** $-4$

**3.** $\int (x^6 + \frac{1}{x^5})\, dx$ is

**A.** $\frac{1}{7}x^7 - \frac{1}{6x^6} + C$

**B.** $\frac{1}{7}x^7 - \frac{1}{4x^4} + C$

**C.** $6x^5 - \frac{5}{x^6} + C$

**D.** $\frac{1}{7}x^7 + \frac{1}{5x^4} + C$

**4.** A parabola has equation $y = x^2 + 8x - 2$. The minimum turning point of this parabola is

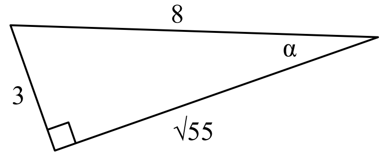
**A.** $(4, 2)$

**B.** $(-4, 2)$

**C.** $(4, -18)$

**D.** $(-4, -18)$

**5.** The diagram shows a right-angled triangle with sides as shown.




The exact value of $\cos 2\alpha$ is

**A.** $\frac{51}{4}$

**B.** $\frac{\sqrt{55}}{4}$

**C.** $\frac{23}{32}$

**D.** $-\frac{23}{32}$

**6.** $f(x) = 4x^3 - 3x^{-\frac{1}{3}}$, $f\,'(x)$ equals

**A.** $x^4 - \frac{9}{2}x^{\frac{2}{3}}$

**B.** $12x^2 + x^{-\frac{4}{3}}$

**C.** $4x^2 - 3x^{-\frac{4}{3}}$

**D.** $12x^2 + x^{\frac{2}{3}}$

**7.** Triangle ABC has vertices A(7, 5), B$(-1,1)$ and C$(-3,4)$. CM is a median of the triangle.

The coordinates of M are

**A.** (4, 3)

**B.** (3, 3)

**C.** (6, 0)

**D.** (4, 2)

**8.** If $(x-3)$ is a factor of $x^3 - 6x^2 + px - 6$, the value of $p$ is

**A.** 0

**B.** 29

**C.** 11

**D.** $-29$

**9.** The equation $2x^2 + x - p = 0$ has no real roots. The range of values of $p$ is

**A.** $p < -8$

**B.** $p < -\frac{1}{8}$

**C.** $p < \frac{1}{8}$

**D.** $p > 8$

**10.** A line has equation $2x - 3y = 4$. The gradient of it is

**A.** 2

**B.** $-2$

**C.** $\frac{2}{3}$

**D.** $-\frac{2}{3}$

**11.** A circle has equation $x^2 + y^2 + 6x - 4y - 3 = 0$. The radius of this circle is

**A.** 7

**B.** $\sqrt{10}$

**C.** $\sqrt{55}$

**D.** 4

**12.** A function $f$ is defined as $f(x) = \frac{1}{1 - x^2}$. The value(s) of $x$ for which this function in undefined is/are

**A.** 0

**B.** 1

**C.** $-1$

**D.** $-1$ and 1

**13.** Here are 4 words which can be used to describe the roots of a quadratic equation

(1) Real

(2) Equal

(3) Unequal

(4) Not real

Which of these words describe the roots of the quadratic equation $(x-2)^2 = 13$?

**A.** (1) and (2)

**B.** (1) and (3)

**C.** (4) only

**D.** some other combination

**14.** The minimum value of $6\sin(x - \frac{\pi}{6}) + 4$ is

**A.** 10

**B.** 2

**C.** $-2$

**D.** $-6$

**15.** When $2x^2 - 16x + 11$ is written in the form $2(x+p)^2 + r$, the value of $p$ is

**A.** $-8$

**B.** 8

**C.** 4

**D.** $-4$

**16.** The line with equation $y = k$ intersects the circle with equation $x^2 + y^2 = 9$ in at least 1 point.

The range of values of $k$ is

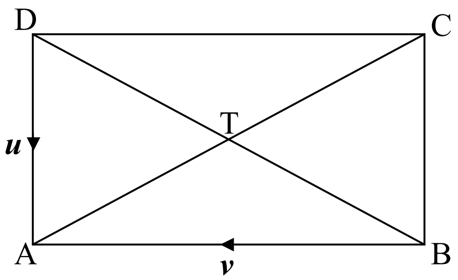
**A.** $-3 < k < 3$

**B.** $-9 < k < 9$

**C.** $-3 \leq k \leq 3$

**D.** $-9 \leq k \leq 9$

**17.**




ABCD is a rectangle. $\overrightarrow{DA} = \boldsymbol{u}$ and $\overrightarrow{BA} = \boldsymbol{v}$

$\overrightarrow{DT}$ equals

**A.** $\frac{1}{2}(\boldsymbol{u}+\boldsymbol{v})$

**B.** $\boldsymbol{u}-\boldsymbol{v}$

**C.** $\frac{1}{2}(\boldsymbol{u}-\boldsymbol{v})$

**D.** $(\boldsymbol{u}+\boldsymbol{v})$

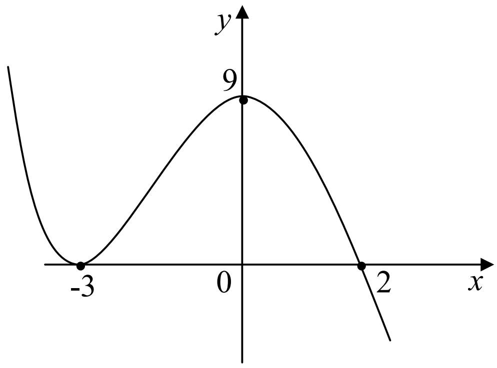
**18.** The vector $\boldsymbol{v}$ is given by $\boldsymbol{v} = \frac{1}{3}\boldsymbol{i} + p\boldsymbol{j}$ where $p > 0$. If $\boldsymbol{v}$ is a unit vector, the value of $p$ is

**A.** $\frac{2\sqrt{2}}{3}$ **B.** $\frac{2}{3}$

**C.** $\frac{\sqrt{10}}{3}$ **D.** $\frac{2\sqrt{2}}{9}$

**19.** The diagram (which is not drawn to scale) shows part of the graph of a cubic function.

The equation of the graph is




**A.** $y = 2(x+3)^2(x-2)$

**B.** $y = -2(x+3)^2(x-2)$

**C.** $y = \frac{1}{2}(x-3)^2(x+2)$

**D.** $y = -\frac{1}{2}(x+3)^2(x-2)$

**20.** The function $g$ is defined by $g(x) = 3x^2 - x^4$ where $x$ is a real number.

The rate of change of $g$ with respect to $x$ at $x = -1$ is

**A.** $-10$

**B.** $2$

**C.** $-2$

**D.** $1$

**Section B**

**21.** Solve algebraically the equation

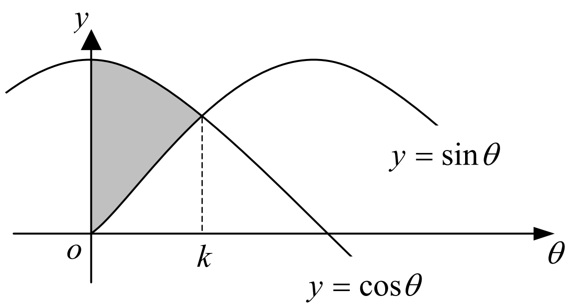
$$\cos 2x^\circ \; - \; 6\sin x^\circ \; + \; 7 \; = \; 0 \qquad \text{for} \qquad 0 \leq x < 360\,. \qquad \textbf{(5)}$$

**22.** A function is defined as $f(x) = \dfrac{3p}{x^2 - 18x + 87}$, for $x \in R$ and $p$ is a constant.

(a) Express the function in the form $f(x) = \dfrac{3p}{(x-a)^2 + b}$, and hence state the maximum value of $f$ in terms of $p$. **(4)**

(b) Given now that $p = \dfrac{2}{\sqrt{2} + 1}$ show that the **exact** maximum value of $f$ is $\sqrt{2} - 1$. **(2)**

**23.** The sketch below shows part of the graphs of $y = \sin\theta$ and $y = \cos\theta$.




(*a*) Write down the value of $k$ in radians. **(1)**

(*b*) Hence show that the exact area of the shaded region is $\sqrt{2} - 1$ square units. **(5)**

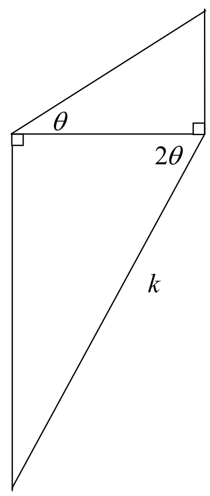
**24.** A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$ , where $a$ and $b$ are constants.

($a$) Given that $U_0 = 4$ and $b = -8$ , express $U_2$ in terms of $a$ . **(2)**

($b$) Hence find the value of $a$ when $U_2 = 88$ and $a > 0$. **(3)**

(c) Given that $S_3 = U_1 + U_2 + U_3$ , calculate the value of $S_3$ . **(2)**

**25.** Consider the diagram opposite where both $\theta$ and $2\theta$ are acute.




($a$) Given that $\tan\theta = \frac{1}{\sqrt{2}}$ , find the **exact** value of

i) $\cos\theta$

ii) $\cos 2\theta$ . **(4)**

($b$) Hence find the **exact** value of $k$ . **(2)**

[ END OF QUESTION PAPER ]