Practice Paper C
Marking Scheme - Paper I Section A

1.
$$\begin{array}{r|rrrr} 3 & 4 & 0 & 3 & -2 \\ & & 12 & 36 & 117 \\ \hline & 4 & 12 & 39 & 115 \end{array}$$

Answer: B

2.
$$3y = x + 6$$
$$y = \frac{1}{3}x + 2$$
$$m = \frac{1}{3}$$
$$m_{perp} = -3$$

Answer: A

3. $2^2 + 1^2 - 4(2) + 6(1) - 3 = 0$

statement (1) is correct

centre is $(2, -3)$

statement (2) is correct

$r^2 = 4 + 9 + 3 = 16$

$r = 4$

statement (3) is incorrect

Answer: A

4.
$$f(x) = 2x^{-4}$$
$$f'(x) = -8x^{-5}$$
$$= \frac{-8}{x^5}$$

Answer: C

5.
$$f(g(x)) = f(3 - x)$$
$$= (3 - x)^2 - 3$$
$$= 9 - 6x + x^2 - 3$$
$$= 6 - 6x + x^2$$

Answer: C

6. *Answer: A*

7.
$$y = x^2 + 3$$
$$7 = x^2 + 3$$
$$x^2 = 4$$
$$x = 2$$

P(2, 7) and Q(1, 4)

$$m_{PQ} = \frac{7-4}{2-1} = 3$$

Answer: A

8. *Answer: C*

9.
$$\cos 2x° = 2 \times \left(\frac{\sqrt{3}}{2}\right) - 1$$
$$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$$

Answer: B

10.
$$\frac{dy}{dx} = 3x^2$$
$$x = -1$$
$$\frac{dy}{dx} = 3(-1)^2 = 3$$

Answer: B

11.
$$\tan \frac{5\pi}{6} (150°) = -\tan \frac{\pi}{6} (30°)$$
$$= -\frac{1}{\sqrt{3}}$$

Answer: C

12.
$$-2 - 12 + 4z = 0$$
$$4z = 14$$
$$z = \frac{14}{4} = \frac{7}{2}$$

Answer: A

13.

$$-\cos 3x \times \frac{1}{3} + C$$

$$= -\frac{1}{3}\cos 3x + C$$

Answer: D

14.

$$L = \frac{-21}{1-0\cdot 3} = \frac{-21}{0\cdot 7} = \frac{-21}{\frac{7}{10}}$$

$$= -21 \times \frac{10}{7} = -30$$

Answer: B

15. $3 + 2 = 5$

Answer: A

16.

$$k = \sqrt{(-1)^2 + 3^3} = \sqrt{10}$$

$$\tan\alpha = \frac{-1}{3}$$

S	A*
* T	C * *

Quadrant IV

Answer: B

17.

$$\frac{d}{dx} 2(x^2+1)^{-1}$$

$$= -2(x^2+1)^{-2}.2x$$

$$= -\frac{4x}{(x^2+1)^2}$$

Answer: C

18. At $x = 0$ graph is decreasing

so $f\,'(0) < 0$ (1) is correct

At $x = 4$ graph is decreasing

so $f\,'(4) < 0$ (2) is incorrect

At $x =$ 5 graph is stationary

so $f\,'(5) = 0$ (3) is correct

Answer: B

19.

$$x^2 - 2kx + k = 0$$

$$a = 1;\ b = -2k;\ c = k$$

$$b^2 - 4ac = 4k^2 - 4.1.k = 0$$

$$4k^2 - 4k = 0$$

$$4k(k-1) = 0$$

$$k = 1$$

Answer: A

20.

$$f\,'(x) = 2ax - 3 = 0$$

$$x = -2$$

$$-4a - 3 = 0$$

$$-4a = 3$$

$$a = -\frac{3}{4}$$

Answer: A

Practice Paper C - Paper 1 Section B Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
21.	**ans:** $f'(x) = 1 + \frac{1}{2}x^{-3/2}$ **4 marks**	
	•¹ for dealing with the denominator	•¹ $f(x) = x^{-1}(x^2 - \sqrt{x})$
	•² correct preparation	•² $= x - x^{-\frac{1}{2}}$
	•³ for diff. 1st term	•³ $f'(x) = 1 + ...$
	•⁴ for diff. 2nd term	•⁴ $= .... + \frac{1}{2}x^{-3/2}$
22a	**ans:** $y = 9x - 5$ **4 marks**	
	•¹ establishing coords. of R a)	•¹ R(0,-5)
	•² strategy of differentiation for m	•² $f'(x) = 3x^2 - 12x + 9 = m$ (stated or implied)
	•³ finding m	•³ $f'(0) = 9 = m$
	•⁴ finding equation of tangent	•⁴ $y = 9x - 5$
b	**ans:** A(1,0) , B(3,0) , C(2,-3) **4 marks**	
	•¹ for knowing to relate S.P's to x-axis	•¹ attempting to find S.P.'s for roots
	•² solving derivative to zero	•² $3x^2 - 12x + 9 = 0$
	•³ for A and B ($x = 1$ or $x = 3$ is o.k.)	•³ $x = 1$ *or* $x = 3$ $\therefore$ A(1,0) , B(3,0)
	•⁴ for C	•⁴ sub. 2 in deriv. $\Rightarrow$ C(2,-3)
23a	**ans:** $\vec{AC} = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$ **1 mark**	
	•¹ for answer	•¹ $\underset{\sim}{c} - \underset{\sim}{a} = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$
b	**ans:** proof **4 marks**	
	•¹ for establishing coords. of B	•¹ B(8,4,-2)
	•² for establishing coords. of D	•² D(6,10,-5)
	•³ for $\vec{CD}$ in component form	•³ $\vec{CD} = c - d = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$
	•⁴ ans.(common point should be mentioned)	•⁴ since $\vec{AC} = 2\vec{CD}$,and C is a common point, then A, C and D are collinear

	Give 1 mark for each •	Illustration(s) for awarding each mark
24a	**ans:** proof **2 marks**	
	•1 equating volumes	•1 $4(x+2)^2 = c(5+8x)$
	•2 expanding and rearranging as required	•2 $4x^2 + 16x - 8cx + 16 - 5c = 0$ to ans.
b	**ans:** $c = \frac{11}{4}$ cm **4 marks**	
	•1 understanding procedure	•1 $b^2 - 4ac = 0$ (stated or implied)
	•2 for selecting and sub. a, b and c	•2 $a = 4,\ b = 16 - 8c,\ c = 16 - 5c$
	•3 expanding and arranging	•3 $64c^2 - 176c = 0$
	•4 factorising to answer	•4 $16c(4c - 11) = 0 \Rightarrow c = \frac{11}{4}$, ($c \neq 0$)
25a	**ans:** $k = -1$ **3 marks**	
	•1 attempting to set up synth. div.	•1 $\begin{array}{r\|rrrrr} 3 & -1 & 4 & -4 & 0 & 9 \\ \hline & & & & & 0 \end{array}$
	•2 for finding quotient	•2 -1 1 -1 -3
	•3 for finding factor of quotient	•3 $\begin{array}{r\|rrrr} -1 & -1 & 1 & -1 & -3 \\ & & 1 & -2 & 3 \\ \hline & -1 & 2 & -3 & 0 \end{array}$ $\therefore k = -1$
b	**ans:** $29\frac{13}{15}$ units^2 **4 marks**	
	•1 setting up correct integral	•1 $\int_{-1}^{3} 9 - 4x^2 + 4x^3 - x^4 \; dx$
	•2 integrating	•2 $\left[9x - \frac{4x^3}{3} + x^4 - \frac{x^5}{5}\right]_{-1}^{3}$
	•3 attempting to substitute correctly	•3 (..........) – (...........)
	•4 calculating the correct answer	•4 $(23\frac{2}{5}) - (-6\frac{7}{15}) = 29\frac{13}{15}$

Total 30 marks

Practice Paper C - Paper 2 Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans: C(3,5)** **1 mark**	
	•[1] for extracting centre	•[1] C(3,5)
b	**ans: B(6,2)** **1 mark**	
	•[1] establishing coords. of B	•[1] B(6,2)
c	**ans:** $y = x - 4$ **3 marks**	
	•[1] for gradient of CB (or equiv.)	•[1] $m = \frac{2-5}{6-3} = -1$
	•[2] knowing $m_1 \times m_2 = -1$, and $m_{\tan}$	•[2] $\therefore\ m_{\tan} = 1$
	•[3] for equation	•[3] $y - 2 = 1(x - 6)$
d	**ans: T(2,-2)** **4 marks**	
	•[1] setting up a system	•[1] solve $\begin{array}{l} x^2 + y^2 = 8 \\ y = x - 4 \end{array}$
	•[2] solving system correctly	•[2] $2(x-2)^2 = 0 \quad \therefore\ x = 2$ (twice)
	•[3] stating 1 root (1 ans.) = a tangent	•[3] written statement (1 ans., 1 point)
	•[4] completing point T	•[4] $y = 2 - 4\ \therefore\ y = -2$, T(2,-2)
2.	**ans:** $\{75\cdot5^\circ, 120^\circ, 240^\circ, 284\cdot5^\circ\}$ **5 marks**	
	•[1] correct double angle sub.	•[1] $4(2\cos^2 x - 1) + 2\cos x + 3 = 0$
	•[2] manipulation to factorising	•[2] $(4\cos x - 1)(2\cos x + 1) = 0$
	•[3] first angle from first factor	•[3] $x = 75\cdot5^\circ$
	•[4] first angle from second factor	•[4] $x = 120^\circ$
	•[5] remaining two angles	•[5] $x = 240^\circ$, $284\cdot5^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3a	**ans:** 4 days **3 marks**	
	•1 setting up recurrence	•1 $U_1 = 0.85(45) + 8$
	•2 knowing to look at <u>low</u> value (before +8)	•2 $U_1 = 0\cdot 85(45) = 38\cdot 25 + 8 = 46\cdot 25$
	•3 calculations and answer	•3 $U_4 = 0\cdot 85(48\cdot 21) = 40\cdot 98 + 8 = 48\cdot 98$ next day low value will be > 41.
b	**ans:** Yes (+ reasons from limits) **3 marks**	
	•1 stating why limit exists	•1 limit exists because $-1 < a < 1$
	•2 calculating limit	•2 $L = \dfrac{b}{1-a} = 53\frac{1}{3}$ (or equiv.)
	•3 considering upper and lower limit in conclusion (own discretion)	•3 solution will always have a strength of between $45\frac{1}{3}$ and $53\frac{1}{3}$ g/gallon.
4a	**ans:** $T_1(-1,-2)$, $T_2(1,-6)$ **4 marks**	
	•1 knowing to differentiate	•1 for S.P.'s $\dfrac{dy}{dx} = 0$ (stated or implied)
	•2 differentiating	•2 $\dfrac{dy}{dx} = 3x^2 - 3$
	•3 solving for x coords.	•3 $3(x^2 - 1) = 0 \quad \therefore \quad x = \pm 1$
	•4 completing points	•4 $T_1(-1,-2)$, $T_2(1,-6)$
b	**ans:** A(-2,-6) , B(2,-2) **4 marks**	
	•1 attempting to solve for x	•1 for A .... $x^3 - 3x - 4 = -6$, etc
	•2 using synth. div. (or trial & error) for A	•2 for A -2 | 1 0 -3 2
	•3 using synth. div. (or trial & error) for B	•3 for B 2 | 1 0 -3 -2
	•4 completing points	•4 A(-2,-6) , B(2,-2)
c	**ans:** $m_1 = m_2 = 9$ $\therefore$ parallel **2 marks**	
	•1 for sub. x values into derivative	•1 @ A , $m = 3(-2^2) - 3 = 9$ @ B , $m = 3(2^2) - 3 = 9$
	•2 statement equal gradients are parallel	•2 since gradients are equal the two tangents are parallel

	Give 1 mark for each •	Illustration(s) for awarding each mark
5a	**ans:** proof **2 marks**	
	•1 correct substitution	•1 $f(g(x)) = \dfrac{1}{\frac{1}{2}(2x^2-4)+1}$
	•2 manipulation to answer	•2 $\ldots = \dfrac{1}{x^2-2+1} = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$
b	**ans:** $x \neq \pm 1$ **1 mark**	
	•1 answer	•1 $x \neq \pm 1$
c	**ans:** proof **4 marks**	
	•1 equating functions	•1 $\dfrac{1}{x^2-1} = \dfrac{1}{\frac{1}{2}x-1}$
	•2 manipulation to quadratic	•2 $\frac{1}{2}x-1 = x^2-1 \Rightarrow x-2 = 2x^2-2$ $\Rightarrow 2x^2-x-4=0$
	•3 use of discriminant (or equiv.)	•3 $b^2-4ac = 1-(4(2)(-4)) = 33$
	•4 statement/conclusion	•4 roots are real, <u>distinct</u> and <u>irrational</u> (or equivalent explanation)
6a	**ans: P(1,3) , Q(3,3)** **2 marks**	
	•1 for equating	•1 $4x-x^2=3$ (or equivalent)
	•2 solving and stating points	•2 $x^2-4x+3=0 \Rightarrow x=1 \text{ or } x=3$ $\Rightarrow$ P(1,3) , Q(3,3)
b	**ans:** $1\frac{1}{3}$ units^2 **5 marks**	
	•1 setting up integral	•1 $\int [(4x-x^2)-3]\,dx$
	•2 for limits	•2 $\int_1^3$ …………
	•3 integrating	•3 $\left[2x^2-\dfrac{x^3}{3}-3x\right]_1^3$
	•4 for subst. Numbers	•4 $(18-9-9)-(2-\frac{1}{2}-3)$
	•5 calculating answer	•5 $1\frac{1}{3}$
c	**ans:** $9\frac{1}{3}$ units^2 **3 marks**	
	•1 finding root for limit	•1 $4x-x^2 = x(4-x)=0 \therefore x=4$
	•2 calc. area between curve and x-axis	•2 $\int_0^4 4x-x^2 = 10\frac{2}{3}$
	•3 subtracting for answer	•3 $10\frac{2}{3}-1\frac{1}{3} = 9\frac{1}{3}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7a	**ans:** $h = 18 - 2r$ **1 mark**	
	•[1] answer	•[1] $d + h = 18 \Rightarrow 2r + h = 18$ $\therefore\ h = 18 - 2r$
b	**ans: proof** **2 marks**	
	•[1] knowing to substitute for h •[2] processing to answer	•[1] $V = \frac{1}{3}\pi r^2(18 - 2r)$ •[2] $V = 6\pi r^2 - \frac{2}{3}\pi r^3$
c	**ans:** $r = 6cm\ ,\ V = 72\pi\ or\ 226\cdot1\ \text{cm}^3$ **5 marks**	
	•[1] method (differentiation) •[2] differentiation •[3] solving for r •[4] proving a maximum (nature table) •[5] calculating V (multiple of π or not)	•[1] $V'(r) = 0$ *at* max. (stated or implied) •[2] $V'(r) = 12\pi r - 2\pi r^2$ •[3] $2\pi r(6 - r) = 0\ \ \therefore\ r = 6\ ,\ r = 0$ •[4] nature table showing a maximum •[5] $V(6) = 216\pi - 144\pi = 72\pi\ \text{cm}^3$
8.	**ans:** $75\cdot9^\circ$ **5 marks**	
	•[1] for dealing with unit vector notation	•[1] $F_1 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}\ ,\ F_2 = \begin{pmatrix} \sqrt{3} \\ 0 \\ 1 \end{pmatrix}$
	•[2] magnitude of F_1 •[3] magnitude of F_2 •[4] for scalar product •[5] for answer	•[2] $F_1 = \sqrt{4+1+4} = 3$ •[3] $F_2 = \sqrt{3+1} = 2$ •[4] $F_1 \,.\, F_2 = 2\sqrt{3} + 0 - 2$ •[5] $75\cdot9^\circ$

Total 60 marks