NATIONAL QUALIFICATIONS

Mathematics

Practice Paper D

Paper 2

Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\quad a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2 \sin A \cos A
\end{aligned}$$

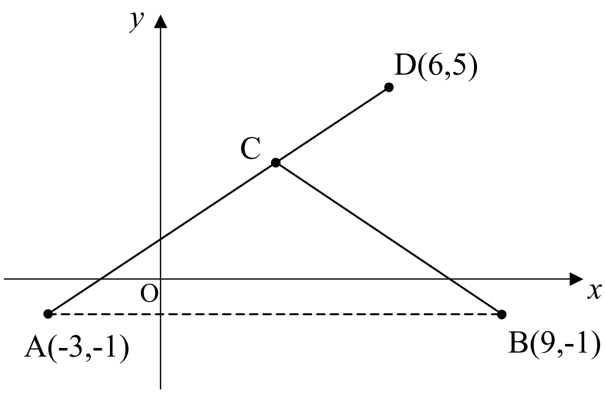
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1. The diagram shows a line joining the points A(-3,-1) and D(6,5).

B has coordinates (9,-1) and C is a point on AD.



(a) Find the equation of the line AD. **2**

(b) Hence establish the coordinates of C given that triangle ABC is isosceles. **3**

(c) Use gradient theory to calculate the size of angle BCD, giving your answer correct to the nearest degree. **3**

2. Solve algebraically the equation

$$\sin 30t^\circ + \sqrt{3}\cos 30t^\circ + 3 = 2, \text{ where } 0 \le t < 12.$$ **6**

3. Three functions are defined on suitable domains as

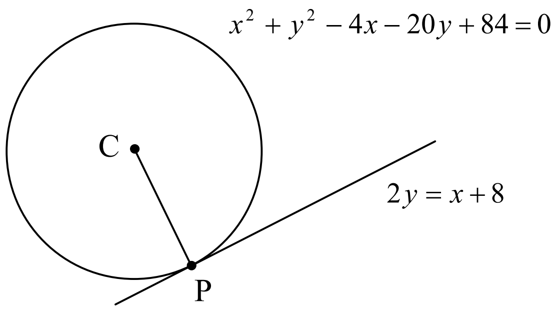
$$f(x) = x - 1, \quad g(x) = 3x^2 - 3 \text{ and } h(x) = x^3 - 6x.$$

(a) Given that $y = g(f(x)) - h(x)$, find a formula for y in its simplest form. **3**

(b) Hence find the coordinates of the maximum turning point of the graph of $y = g(f(x)) - h(x)$, **justifying your answer**. **4**

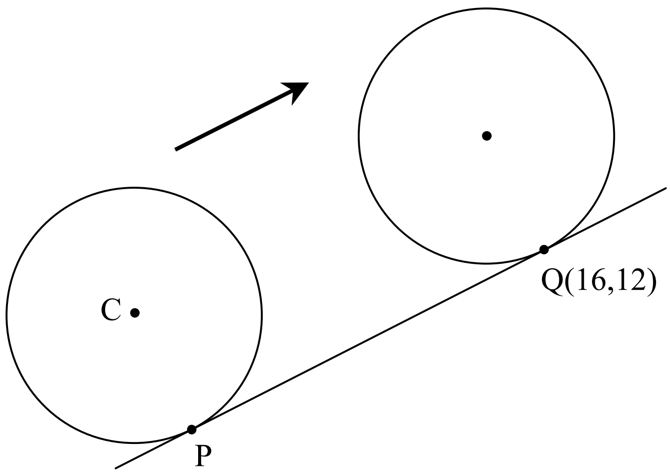
4. A circle, centre C, has as its equation $x^2 + y^2 - 4x - 20y + 84 = 0$.

It touches the line with equation $2y = x + 8$ at point P, as shown.



(a) Find **algebraically** the coordinates of P. **4**

(b) The circle is rolled up the line until Q(16,12) becomes the new point of tangency.



Establish the equation of the circle in this new position. **5**

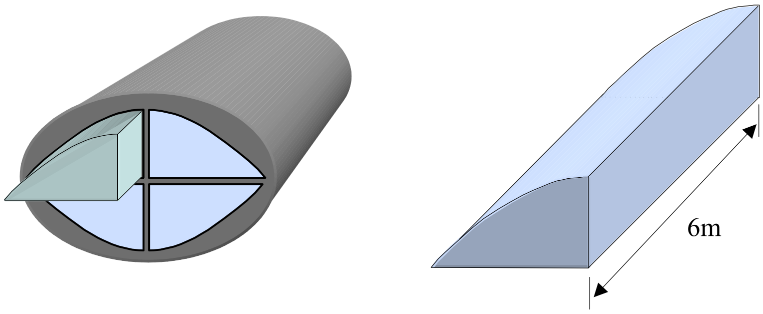
5. The angle θ is such that $\tan\theta = \frac{2}{\sqrt{2}}$ where $0 < \theta < \frac{\pi}{2}$.

(a) Find the exact values of $\sin\theta$ and $\cos\theta$. **3**

(b) Hence show clearly that the exact value of $\sin(\theta + \frac{\pi}{3})$ can be expressed as

$$\sin(\theta + \tfrac{\pi}{3}) = \tfrac{1}{6}(\sqrt{6} + 3).$$ **5**

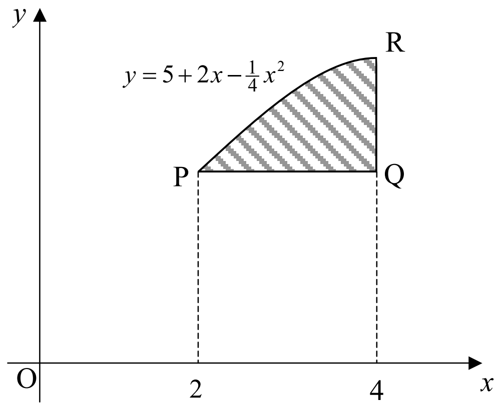
6. A titanium rod from a nuclear reactor is a solid prism which slots into an elliptical chamber along with three other identical rods. It has a cross-sectional shape made up of two straight lines and a curved edge.



Each rod has a depth of 6 metres.

The cross section of a rod is shown geometrically in the coordinate diagram below where the **units are in metres**. The diagram is not drawn to scale.

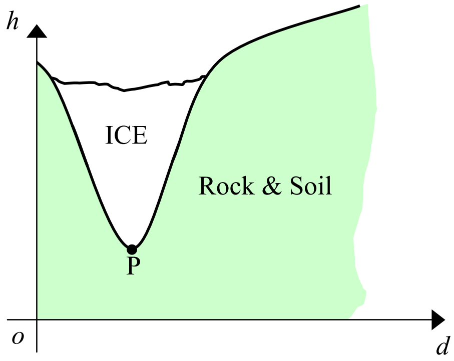
The curved section is part of the graph of the curve with equation $y = 5 + 2x - \frac{1}{4}x^2$.
PQ is horizontal and QR is vertical.



(a) Calculate the shaded area in square metres. **7**

(b) Hence calculate the **total volume** of titanium contained in **all four rods**. **2**

7. The graph below shows the cross section of a small glacier.
The horizontal axis indicates the amount of level drift, d metres, and has a scale of 1 unit represents 150 metres.
The vertical axis is the approximate height, h metres, above sea level and has a scale of 1 unit represents 100 metres.



(*a*) The curved lower edge of the glacier is found to be the function defined as

$$h(d) = \left[\frac{-4}{d^2 - 4d + 5} \right] + 6 \quad , \quad \text{for} \quad 0 \le d \le 5 \, .$$

Express the function in the form

$$h(d) = \left[\frac{-4}{(d-a)^2 + b} \right] + 6$$ **2**

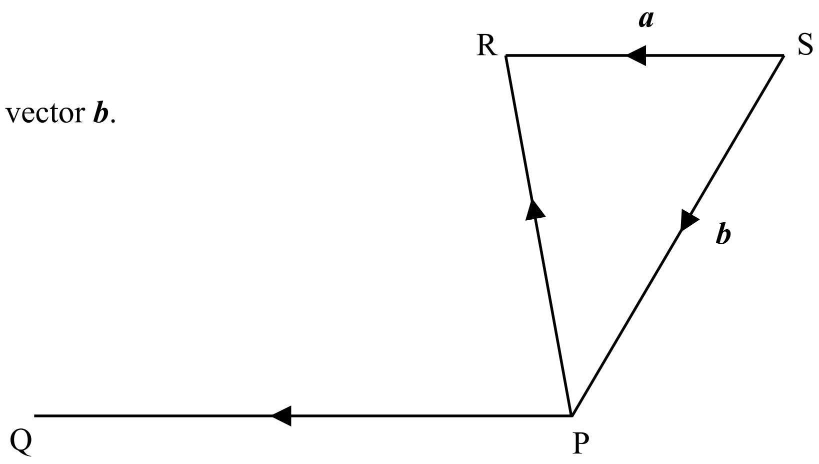
(*b*) Hence state the minimum value of h and the corresponding value of d. **2**

(*c*) With reference to the origin, and using the scales given, state the position of P in metres. **1**

8. Consider the vector diagram.

SR represents vector $\boldsymbol{a}$ and SP vector $\boldsymbol{b}$.

Angle PSR $= 60^{\circ}$.



(a) Express displacement PR in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. **1**

(b) Given that vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ have magnitudes of 2 units and 3 units respectively, evaluate the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$. **2**

(c) Hence evaluate the scalar product $\boldsymbol{v} \cdot \boldsymbol{u}$ when $\boldsymbol{v}$ = PQ = $2\boldsymbol{a}$ and $\boldsymbol{u}$ = PR. **5**

[END OF QUESTION PAPER]