Mathematics

Practice Paper G

Paper 2

Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

NATIONAL
QUALIFICATIONS

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A \cos A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

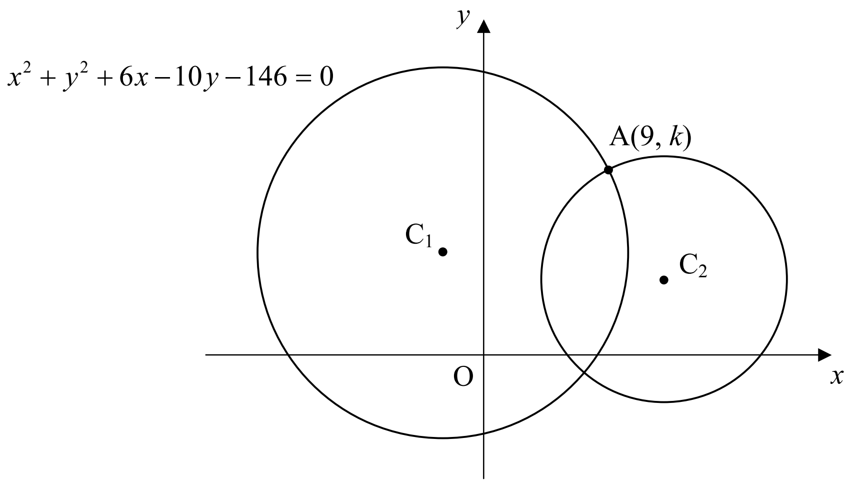
$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Two intersecting circles are shown in the diagram below.

The circle, centre C_1 , has $x^2 + y^2 + 6x - 10y - 146 = 0$ as its equation.

The point A(9, *k*) lies on the circumference of both circles.



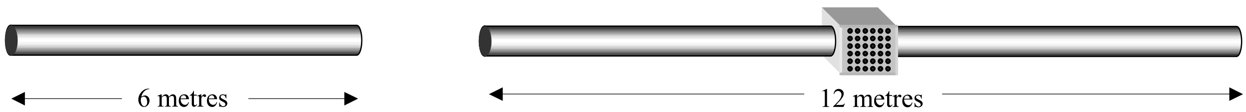
(a) Establish the value of *k*. **2**

(b) The second circle has the point $C_2(p, 3)$ as its centre.

Given that angle C_1AC_2 is a right-angle, find the value of *p*. **5**

(c) Hence find the equation of a third circle which passes through C_1, A and C_2. **4**

2. In a steam turbine the blades are rotated using superheated steam. Superheated steam has many advantages, one being its ability to travel long distances (through tubing) with minimal heat loss. One way of keeping the temperature of the superheated steam as constant as possible is to apply heat, through heated elements, at intervals along the tubing (see diagram).



(a) In a particular turbine superheated steam enters the tubes at a temperature of 1050^{o}F. It is known that the steam loses 2% of its temperature for every metre of tubing travelled.

Calculate the expected temperature of the superheated steam as it leaves a plain 6 metre length of tubing. Give your answer correct to the nearest degree. **3**

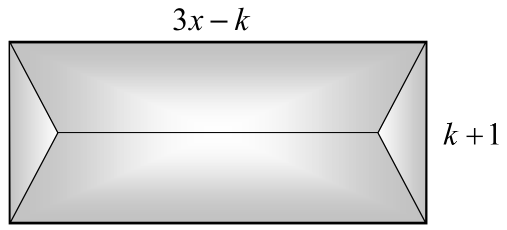
(b) Heating elements are placed every 6 metres but not at the beginning or the end of the tubing. Each of these elements increases the temperature of the steam passing over it by 60^{o}F.

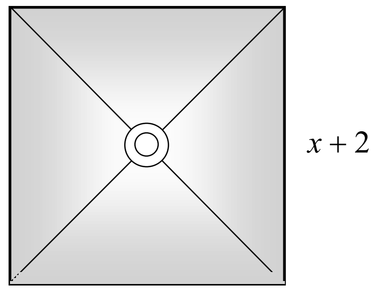
Calculate the temperature of the steam as it leaves a 30 metre section of this tubing. **3**

(c) With this system in place, calculate the approximate temperature of the steam leaving a tube of infinite length. **3**

3. A householder is considering two different designs for a conservatory.

One design has a rectangular base measuring $3x - k$ by $k + 1$ metres and the other design is square based with side $x + 2$ metres. Both x and k are constants.



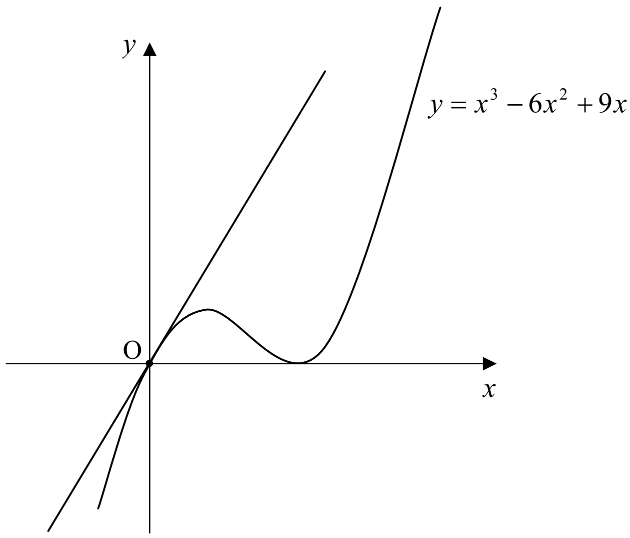


(a) With both designs having the same base **area**, show clearly that the following equation can be formed.

$$x^2 + (1 - 3k)x + (k^2 + k + 4) = 0$$ **4**

(b) Given that the above equation has **equal roots**, find first the value of k, and then the base area of each conservatory in square metres. **5**

4. The diagram, which is not drawn to scale, shows part of the graph of $y = x^3 - 6x^2 + 9x$. The tangent to the curve at the point where $x = 0$ is also drawn.



(a) Establish the equation of the tangent. **3**

(b) This tangent meets the curve at a second point P.

Find the coordinates of P. **4**

5. Two functions are defined on suitable domains as

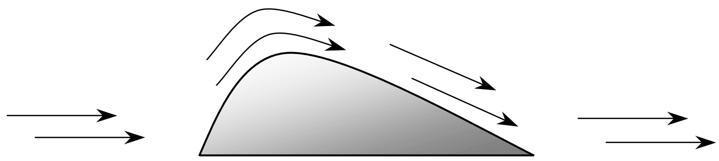
$$f(x) = 2\cos(x)^\circ + 2\sin(x)^\circ \quad \text{and} \quad g(x) = (x)^2.$$

(a) Show that the composite function $g(f(x))$ can be written in the form

$$g(f(x)) = 4(1 + \sin(2x)^\circ).$$ **4**

(b) Hence solve the equation $g(f(x)) = \cos(x)^\circ + 4$ for $0 \le x < 360$. **4**

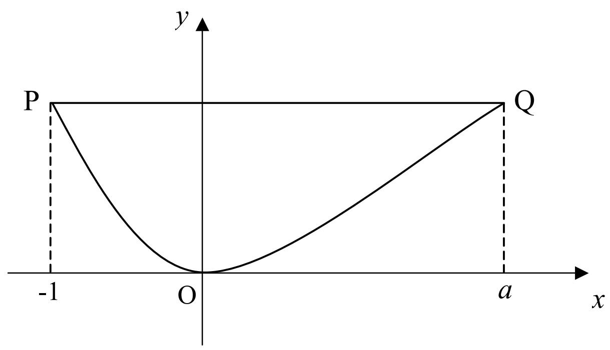
6. A designer is working on a new competition helmet for olympic ski racers. He has perfected the design to produce as little drag and as even a wind-flow over the helmet as possible.



Below is part of his computer aided design showing a flat cross-section of the helmet relative to a set of rectangular axis. The helmet has been rotated through 180°.

The curve PQ has as its equation $y = 3x^2 - x^3$. The line PQ is horizontal.

The x-coordinates of P and Q are -1 and a respectively.



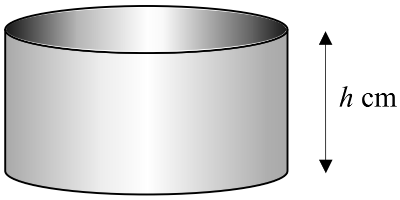
(a) **Show clearly** that the equation of the line PQ is $y = 4$. **1**

(b) Hence determine the value of a. **3**

(c) Calculate the **area** enclosed between the line PQ and the curve with equation $y = 3x^2 - x^3$.
Give your answer in square units. **4**

7. An cylindrical container, open at the top, has a volume of 64π cubic centimetres and a height of h centimetres.

(a) Show that $h = \dfrac{64}{r^2}$. **1**



(b) If the radius of the container is r cm, show that the total surface area, A, of the container, can be represented by the function

$$A(r) = \frac{128\pi}{r} + \pi r^2.$$ **2**

(c) Hence find the dimensions of the cylinder so that this surface area is a minimum. **5**

[END OF QUESTION PAPER]