Mathematics Higher

Practice Papers
for SQA Exams

Exam K
Higher
Paper 2

You are allowed 1 hour, 10 minutes to complete this paper.

You may use a calculator.

Full marks will only be awarded where your answer includes relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Table of standard derivatives

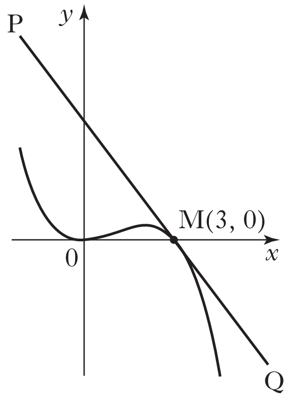
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Marks

1. The diagram shows a cubic curve with equation $y = x^2 - \frac{1}{3}x^3$.



A tangent PQ to the curve has point of contact M(3, 0).

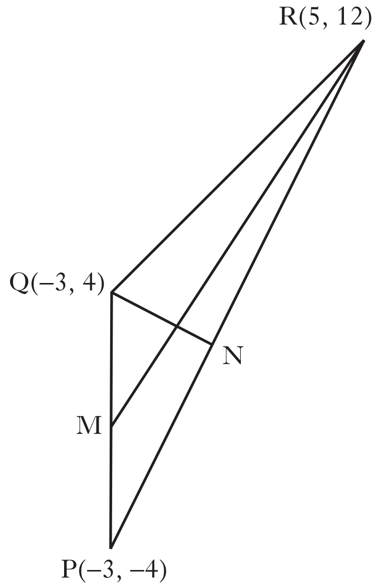
(*a*) Find the equation of PQ **4**

A circle has equation $x^2 + y^2 - 4x - 26y + 163 = 0$

(*b*) Show that PQ is also a tangent to this circle and find the coordinates of the point of contact N **6**

(*c*) Find the ratio in which the y-axis cuts the line MN **3**

2. Triangle PQR has coordinates P(−3, −4), Q(−3, 4) and R(5, 12)



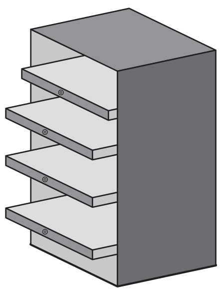
(*a*) Find the equation of the median MR **3**

(*b*) Find the equation of the attitude NQ **3**

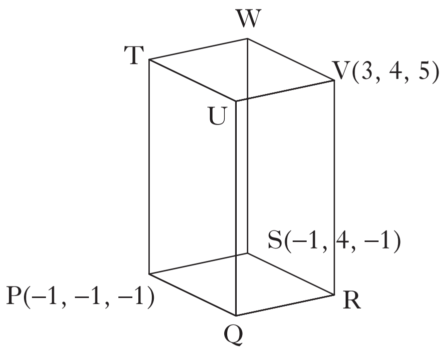
(*c*) Median MR and altitude NQ intersect at point S. Find the coordinates of S. **3**

(*d*) The point T(2, 9) lies on QR. Show that ST is parallel to PR **2**

3.



This set of drawers is being 'modelled' on a computer software design package as a cuboid as shown. The edges of the cuboid are parallel to the x, y and z-axes. Three of the vertices are P(−1,−1,−1), S(−1,4,−1) and V(3,4,5)



(*a*) Write down the lengths of PQ, QR and RV. **1**

(*b*) Write down the components of $\overrightarrow{VS}$ and $\overrightarrow{VP}$ and hence calculate the size of angle PVS. **7**

Marks

4. (*a*) Express $3\cos x° - 2\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a \leq 90$ **4**

(*b*) Hence solve the equation $3\cos x° - 2\sin x° = 2$ for $0 < x < 360$. **3**

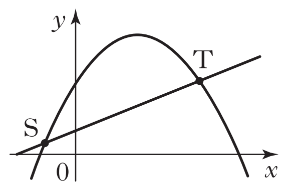
5. Atmospheric pressure decreases exponentially as you rise above sea-level. It is known that the atmospheric pressure, $P(h)$, at a height h kilometres above sea level is given by $P(h) = P_o e^{-kh}$ where P_o is the pressure at sea-level ($h = 0$).

(*a*) Given that at a height of 4.95 km the atmospheric pressure is half that at sea-level, calculate the value of k correct to 4 decimal places. **3**

(*b*) Mount Everest is 8850 metres high. What is the percentage decrease in air pressure at the top of Mount Everest compared to the pressure at sea-level? **2**

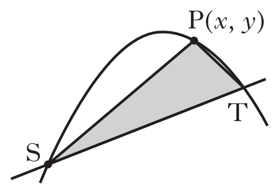
6. The diagram shows the curve with equation $y = 6 + 4x - x^2$ and the straight line with equation $y = x + 2$. The line intersects the curve at points S and T as shown

(*a*) Calculate B unit the exact value of the area enclosed by the curve and the line **7**



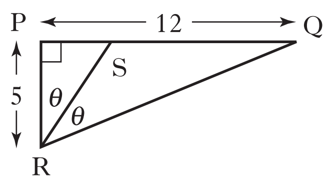
(*b*) A point $P(x, y)$ lies on the curve between S and T and it is known that the area, A, of triangle PST (shaded in the diagram) is given by

$$A(x) = -\frac{5}{2}x^2 + \frac{15}{2}x + 10$$



Calculate the maximum value of this area and hence determine what fraction this maximum value is of the area B unit^2 from part (a). **4**

7. In right-angled triangle PQR, RS is the bisector of angle PRQ. PR = 5 units and PQ = 12 units. Show that the exact value of $\cos\theta$ is $\frac{3\sqrt{13}}{13}$



5

[End of question paper]