Mathematics
Higher
Paper 1
Practice Paper Q

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. The midpoint of the line joining $G(-1, 3, 7)$ to $H(5, -1, p)$ is $M(q, 1, 4)$.

   What are the values of p and q?

	p	q
A	1	2
B	3	2
C	1	–3
D	3	–3

2. Given that $f(x) = \frac{1}{3x^5}$, find $f'(x)$.

   A $-\frac{15}{x^4}$

   B $-\frac{1}{15x^4}$

   C $\frac{1}{15x^4}$

   D $-\frac{5}{3x^6}$

3. If $x^2 + 12x + 7$ is written in the form $(x+a)^2 + r$, find the value of r.

   A –29

   B –5

   C 1

   D 7

4. A straight line passes through the points $(4, 3)$ and $(0, -1)$.

What is the equation of the line?

A $x+y-1=0$

B $x-y-1=0$

C $x-2y-1=0$

D $3x-4y-1=0$

5. Functions f and g are defined on the set of real numbers by

$$f(x)=x^2+1 \text{ and } g(x)=3x-5$$

What is the value of $g(f(-1))$?

A -5

B -4

C 0

D 1

6. The vectors with components $\begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ t \\ -2 \end{pmatrix}$ are perpendicular.

What is the value of t?

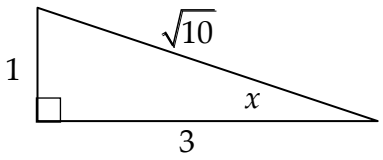
A 2

B 0

C $-\frac{1}{2}$

D -1

7. The diagram shows a right-angled triangle with sides 1, 3 and $\sqrt{10}$.



What is the value of $\cos 2x$?

A $\frac{3}{5}$

B $\frac{4}{5}$

C $\frac{1}{\sqrt{10}}$

D $\frac{2}{\sqrt{10}}$

8. Find $\int_{-2}^{0} 6x^2 \, dx$

A 12

B 16

C 24

D 96

9. For what value of k does the equation $2x^2 - 4x + k = 0$ have equal roots?

A -2

B 0

C 2

D 4

10. $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have components $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$ respectively.

Given that D has coordinates $(-2, 0, -2)$, what are the coordinates of F ?

A $(0, 1, 1)$

B $(1, 3, 0)$

C $(5, 1, 4)$

D $(9, 1, 6)$

11. What is the maximum value of $8 - 3\sin\left(x - \frac{7\pi}{9}\right)$?

A -3

B -1

C 8

D 11

12. Find $\int (2x+5)^3 \, dx$.

A $\frac{1}{2}(2x+5)^3 + c$

B $8(2x+5)^4 + c$

C $\frac{1}{8}(2x+5)^4 + c$

D $(x^2+5x)^4 + c$

13. How many solutions does the equation $(\sqrt{7}\cos x + 3)(4\tan x - 9) = 0$ have in the interval $0 \le x < 2\pi$?

A 0

B 2

C 3

D 4

14. Given that $f(x) = 4\sin 3x$, find $f'\left(\frac{\pi}{6}\right)$.

A –4

B –3

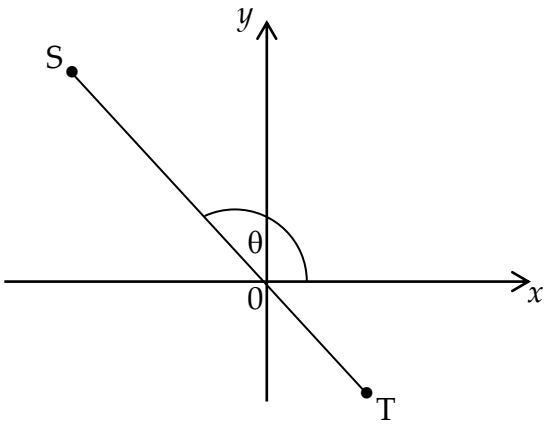
C 0

D 12

15. The diagram shows the line ST with equation $2x + y = 0$.

The angle between ST and the positive direction of the x-axis is θ.

Find an expression for θ.



A $\theta = \tan^{-1}\frac{1}{2}$

B $\theta = \pi - \tan^{-1}\frac{1}{2}$

C $\theta = \tan^{-1} 2$

D $\theta = \pi - \tan^{-1} 2$

16. What is the value of $\dfrac{\log_2 32}{\log_2 8}$?

A $\frac{5}{3}$

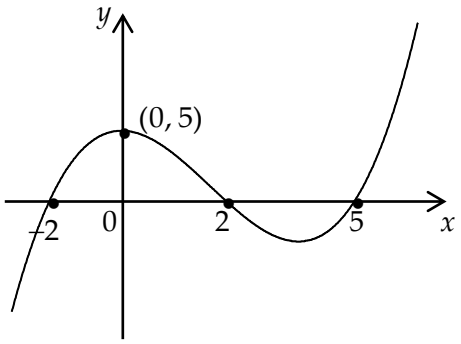
B 2

C 4

D 15

17. The diagram shows a sketch of the curve with equation

$$y = k(x+2)(x-2)(x+a)$$



What are the values of a and k?

	a	k
A	–5	$\frac{1}{4}$
B	–5	–4
C	5	$\frac{1}{4}$
D	5	–4

18. Here are two statements about the function $f(x) = \sqrt{x^2 - 4}$.

(1) The largest possible domain is $-2 \le x \le 2$.

(2) The range is $f(x) \ge 0$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

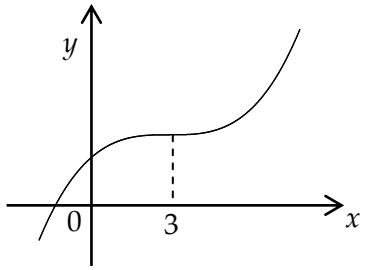
D Both statements are correct.

19. Given that

$$f'(x)\begin{cases} > 0, & \text{for } x < 3 \\ = 0, & \text{for } x = 3 \\ > 0, & \text{for } x > 3 \end{cases}$$

Which diagram shows the curve with equation $y = f(x)$?

A



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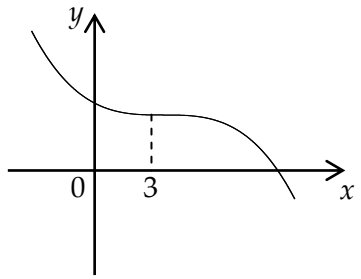

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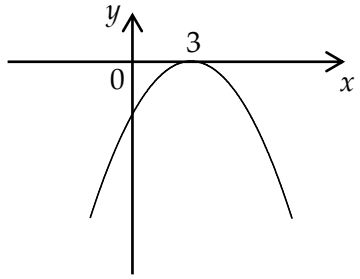

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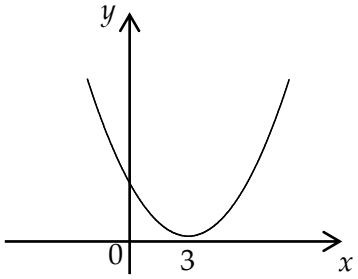


20. If $5^x = a^2$, find an expression for x.

A $x = \frac{a^2}{5}$

B $x = \sqrt[5]{a^2}$

C $x = \frac{2}{\log_a 5}$

D $x = \frac{5}{\log_2 a}$

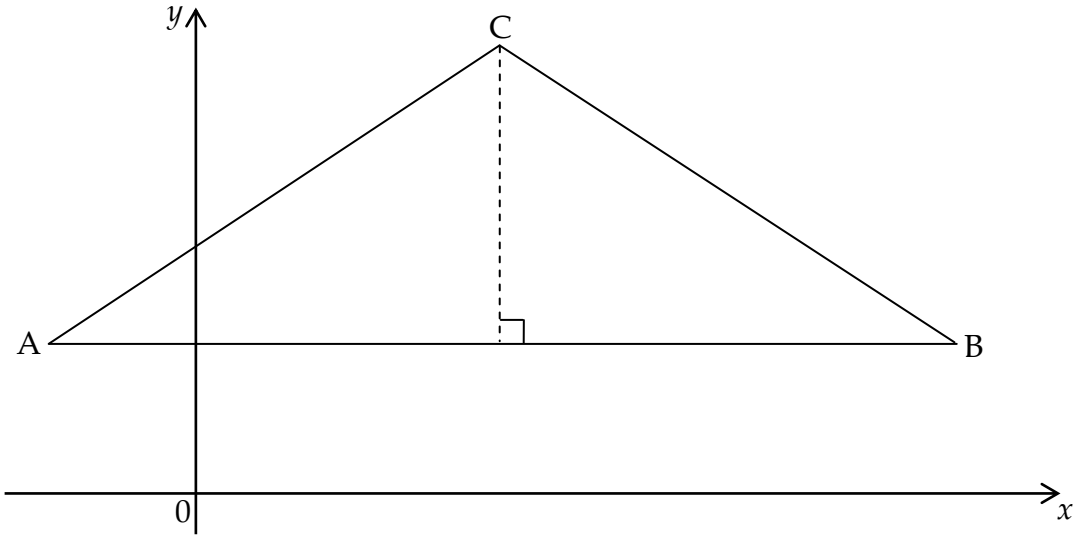
End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. A(−2, 4), B(10, 4) and C(4, 8) are the vertices of triangle ABC shown in the diagram.



(*a*) Write down the equation of the altitude from C. **1**

(*b*) Find the equation of the perpendicular bisector of BC. **4**

(*c*) Find the point of intersection of the lines found in (*a*) and (*b*). **2**

22. P is the point (4, 1, −2), Q is (5, 2, 0) and R is (7, 4, 4).

(*a*) Show that P, Q and R are collinear. **3**

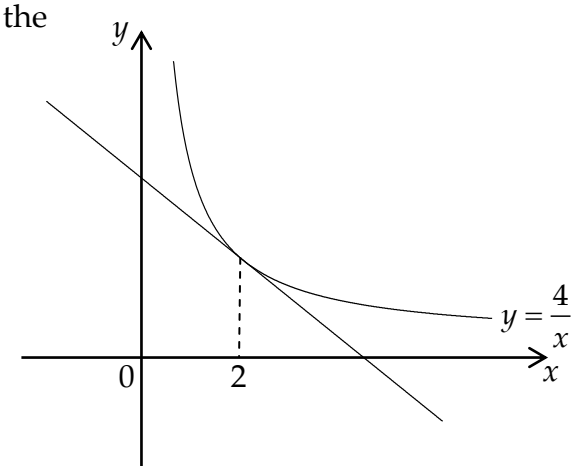
(*b*) Find the ratio in which Q divides PR. **1**

Marks

23. Find the equation of the tangent to the curve with equation

$$y = \frac{4}{x}$$

at the point where $x = 2$.



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24. (*a*) Given that $f'(x) = 3x^2 + 2x - 10$ and $(x-2)$ is a factor of $f(x)$, find a formula for $f(x)$. **4**

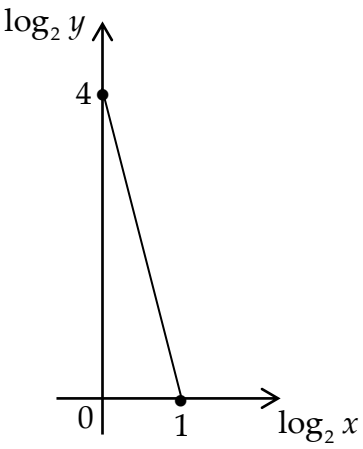
(*b*) Hence factorise $f(x)$ fully. **1**

(*c*) Solve $f(x) = 0$. **1**

25. The graph illustrates the law $y = ax^b$.

The straight line joins the points (0, 4) and (1, 0).

Find the values of a and b.



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End of question paper