

St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 1

Paper II – Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

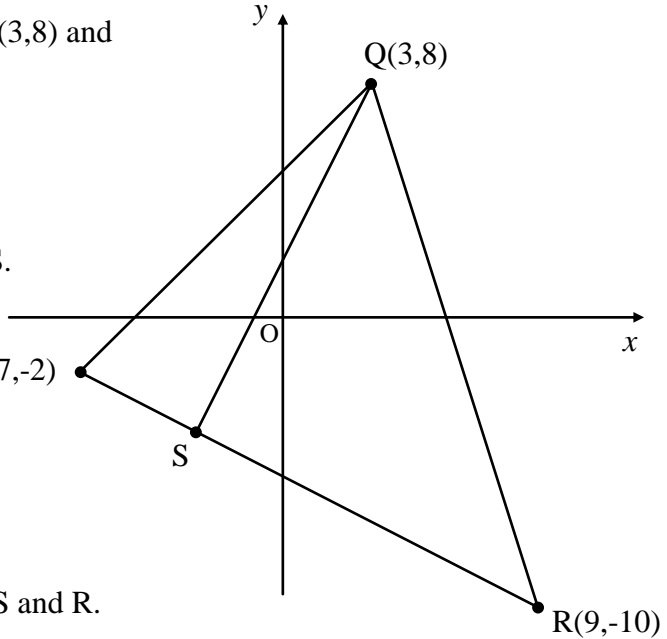
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

All questions should be attempted

1. Triangle PQR has as its vertices $P(-7,-2)$, $Q(3,8)$ and $R(9,-10)$ as shown.



(a) Find the equation of side PR. 2

(b) Find the equation of the **altitude** QS. 3

(c) Hence find the coordinates of S, $P(-7,-2)$ the point where the altitude QS meets side PR. 4

(d) Establish the equation of the circle which passes through the points Q, S and R. 4

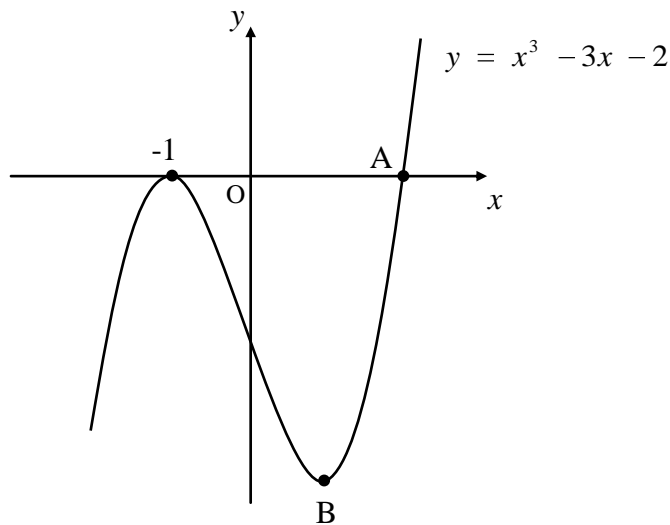
2. A recurrence relation is defined as $u_{n+1} = 0.75u_n + 12$.

Given that $U_0 = 32$, find the **difference** between the limit of the sequence and the third term, U_3 . 5

3. A curve has as its equation $y = (x-6)^2 + 8$.

Given that the line with equation $y = 2x - 5$ is a tangent to this curve, establish the coordinates of the point T, the point of contact between the curve and the line. 4

4. Part of the graph of the curve with equation $y = x^3 - 3x - 2$ is shown below. The curve passes through the point $(-1,0)$.



Find, algebraically, the coordinates of the points A and B. 7

6. The functions f and g , defined on suitable domains, are given as

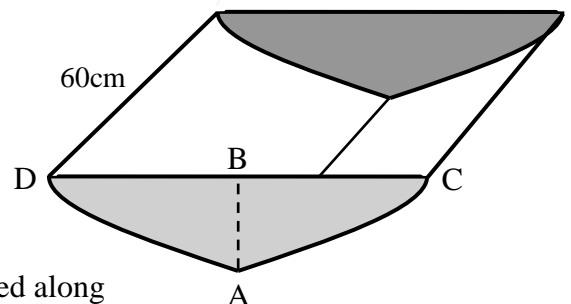
$$f(x) = \frac{x^2}{2} - \frac{3}{4} \quad \text{and} \quad g(x) = \frac{5ax}{4} - a, \quad \text{where } a \text{ is a constant.}$$

- (a) Given that $f(a) = g(1)$, find the value of a , where $a < 0$. 4
- (b) With a taking this value, find the **rate of change** of g . 2

7. A small feeding trough is shown opposite.

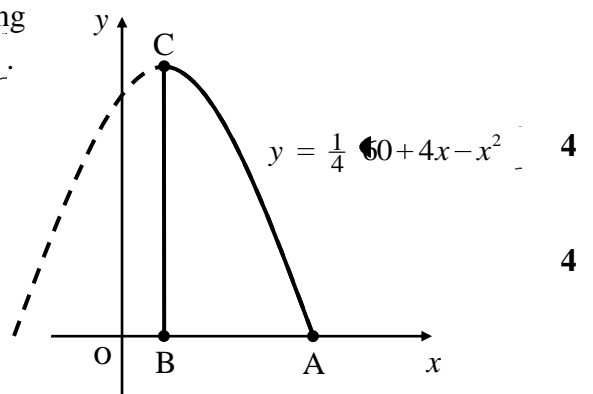
The end face has an axis of symmetry AB.

Edge CD is perpendicular to the axis of symmetry.



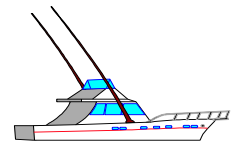
When the end face is rotated through 90° and then halved along the axis of symmetry, shape ABC can be placed on a coordinate diagram as shown below.

AB lies along the x -axis with the curved edge CA being part of the curve with equation $y = \frac{1}{4}(60 + 4x - x^2)$.



- (a) Establish the coordinates of A and B. 4
- (b) Hence calculate the **area** of shape ABC given that all the units are in centimetres. 4
- (c) Given that the trough is a prism and measures 60cm from back to front, calculate the volume of feed the trough can hold when full, giving your answer correct to the nearest litre. 3

9. The captain of a small pleasure boat wishes to take a group of passengers from one island to the next, a journey of 100 kilometres.



The amount of fuel used is dependent upon the speed, v kilometres per hour, of the boat.

- (a) Given that the rate of fuel used is $(1 + 0.0000625 v^3)$ gallons per hour, **show clearly** that the total fuel used, F , for this 100 kilometre journey is given by

$$F = \frac{100}{v} + 0.00625 v^2 \text{ gallons.} \quad \mathbf{2}$$

- (b) Hence find the speed which keeps the amount of fuel used to a minimum and the amount of fuel needed, at this speed, for the voyage. **6**

[END OF QUESTION PAPER]