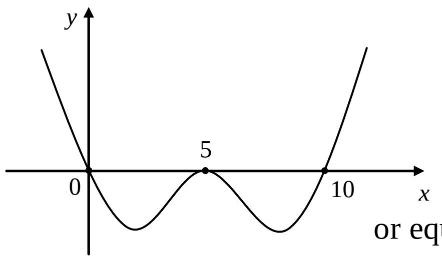
Higher Prelim Revision 2 **Marking Scheme - Paper I**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** M(9,3) **1 mark** •1 answer **(b) ans:** $y = \frac{1}{3}x$ **2 marks** •1 for gradient •2 for strategy **(c) ans:** i) $y = -3x + 60$, C(18,6) ii) congruent **6 marks** i) •1 for gradient of BC •2 for sub. to equ. of line •3 knowing to solve system •4 coordinates of C ii) •5 answer •6 explanation	(a) •1 M(9,3) (b) •1 $m_{AC} = \frac{3-0}{9+0} = \frac{1}{3}$ •2 $y = mx$ $y = \frac{1}{3}x$ (c) i) •1 $m = -3$ •2 $y - 0 = -3(x - 20)$ •3 $\frac{1}{3}x = -3x + 60$ •4 $x = 18 \Rightarrow \therefore y = 6$ ii) •5 congruent (or equiv.) •6 explanation of parallel lines (or any suitable explanation)
2.	**ans:** $m = 2$ **6 marks** •1 for dealing with denominator •2 for simplifying •3 diff. first term •4 diff. second term •5 substituting •6 answer	•1 $y = x^{-\frac{1}{2}}(x^2 - 4x)$ •2 $y = x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$ •3 $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ .... •4 $\frac{dy}{dx} =$ .... $- 2x^{-\frac{1}{2}}$ •5 $m = \frac{3}{2}\sqrt{4} - \frac{2}{\sqrt{4}}$ (or equiv.) •6 $m = 2$
3.	**ans:** see sketch **4 marks** •1 for stat. points as roots •2 for basic shape … left side •3 basic shape … right side •4 annotation	 or equivalent sketch
4.	**ans:** $a = 2$, $b = 1$ **4 marks** •1 sub. for the composite function •2 expanding and simplifying •3 factorising •4 answers	•1 $g(h(x)) = (2x+1)^2 - 3(2x+1)$ •2 $g(h(x)) = 4x^2 - 2x - 2$ •3 $= 2(2x+1)(x-1)$ •4 $a = 2$, $b = 1$

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5.	**(a) ans:** proof **3 marks** •1 for area strategy •2 for substitution •3 for answer **(b) ans:** $\theta = \frac{\pi}{9}$ **3 marks** •1 for strategy and writing .. $\tan 3\theta =$ •2 for knowing exact value •3 calculating answer	(a) •1 $A = \frac{1}{2}bh$ •2 $A = \frac{1}{2}bh = \frac{1}{2} \times x \times 4\sqrt{3}$ •3 $8\sqrt{3} = x \times 2\sqrt{3} \quad \therefore x = 4$ (b) •1 $\tan 3\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$ •2 $If \quad \tan 3\theta = \sqrt{3} \quad then \quad 3\theta = \frac{\pi}{3}$ •3 $\therefore \theta = \frac{\pi}{9}$
6.	**ans:** $a = \frac{2}{3}$ **7 marks** •1 for setting up integral •2 integrating correctly •3 making integral equal 4 •4 substituting •5 simplifying to quadratic equ. •6 factorising •7 solving to answer	•1 $A = \int_1^{1+a} (6x-2)\, dx$ •2 $= \left[3x^2 - 2x \right]_1^{1+a}$ •3 $\left[3x^2 - 2x \right]_1^{1+a} = 4$ •4 $(3(1+a)^2 - 2(1+a)) - (1) = 4$ •5 $3a^2 + 4a - 4 = 0$ •6 $(3a-2)(a+2) = 0$ •7 $\therefore a = \frac{2}{3}$ (note: -2 is a discard)
7.	**(a) ans:** since for both $-1 < a < 1$ **1 mark** •1 for statement **(b) ans:** $b = 5$ **4 marks** •1 know how to find a limit •2 substitute •3 equate both limits •4 solve for *b*	(a) •1 since for both $-1 < a < 1$ (or equiv.) (b) •1 $L = \frac{b}{1-a}$ …. or equivalent •2 $L_1 = \frac{20}{1-0\cdot 6}$, $L_2 = \frac{b}{1-0\cdot 9}$ •3 $\frac{20}{1-0\cdot 6} = \frac{b}{1-0\cdot 9}$ •4 $b = \frac{20 \times 0\cdot 1}{0\cdot 4} = 5$

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8.	**(a) ans:** $x^2+y^2-10y=0$ **4 marks** •1 for radius (5 units) •2 for strategy •3 for substituting in formula •4 for expanding **(b) ans:** $k=2$ **5 marks** •1 knowing to substitute point in equ. •2 simplifying to quadratic •3 solving to answers •4 discarding $k=8$ •5 answer **(c) ans:** $3y=4x-10$ **3 marks** •1 for gradient of radius •2 for gradient of tangent •3 sub. to answer	(a) •1 $r=5$ •2 $(x-a)^2+(y-b)^2=r^2$ •3 $(x-0)^2+(y-5)^2=25$ •4 $x^2+y^2-10y+25-25=0$ .... (b) •1 $4^2+k^2-10k=0$ •2 $k^2-10k+16=0$ •3 $(k-8)(k-2)=0$ •4 ~~$\therefore\ k=8$~~ •5 $k=2$ (c) •1 $m_r=\frac{2-5}{4-0}=-\frac{3}{4}$ •2 $m_{\tan}=\frac{4}{3}$ •3 $y-2=\frac{4}{3}(x-4)$
9.	**(a) ans:** $\frac{dy}{dx}=3(p+1)x^2-6px+4$ **2 marks** •1 differentiating first term •2 differentiating remainder **(b) ans:** $p=2$ **5 marks** •1 realising strategy i.e. equal roots •2 for a, b and c •3 for substitution •4 for simplifying + factorising •5 choosing correct answer	(a) •1 $\frac{dy}{dx}=3(p+1)x^2$ .... (or equiv.) •2 $\frac{dy}{dx}=$ .... $-6px+4$ (b) •1 $b^2-4ac=0$ (stated <u>or</u> implied) •2 $a=3p+3,\ b=-6p,\ c=4$ •3 $(-6p)^2-16(3p+3)=0$ •4 $36p^2-48p-48=0$ $12(3p+2)(p-2)=0$ •5 $\therefore$ ~~$p=-\frac{2}{3}$~~ , $p=2$

Total 60 marks

	Give 1 mark for each •	**Illustration(s) for awarding each mark**
1.	**(a) ans:** $3y = 2x + 3$ **2 marks** •1 for gradient •2 for sub. to answer **(b) ans:** C(3,3) **3 marks** •1 realising mid-point gives $x = 3$ •2 knowing to sub. in equation •3 calculating y correctly then answer **(c) ans:** 67^o **3 marks** •1 for knowing to use $\tan\theta = m$ •2 equating and calculating an angle •3 working towards and finding angle	(a) •1 $m = \frac{5+1}{6+3} = \frac{2}{3}$ •2 $y - 5 = \frac{2}{3}(x - 6)$ (b) •1 $mid_{AB} = \frac{-3+9}{2} = 3$ •2 $\therefore 3y = 2(3) + 3$ •3 $3y = 9 \quad \therefore \ y = 3 \Rightarrow$ C(3,3) (c) •1 $\tan\theta = m$ •2 $\tan D\hat{A}B = \frac{2}{3} \quad \therefore \angle DAB \approx 33\cdot7^o$ •3 working through isosceles triangle then $\angle BCD \approx 67^o$
2.	**(a) ans:** 432 feet **4 marks** •1 knowing to differentiate •2 differentiating and solving to zero •3 finding t for max. height •4 substituting to find height **(b) ans:** No since $28\cdot8 < 48$ ft/s **2 marks** •1 evaluating value of derivative •2 answer + viable explanation **(c) ans:** 420 feet **3 marks** •1 for knowing to solve derivative to 48 •2 calculating t •3 substituting t to answer	(a) •1 max height when $h'(t) = 0$ •2 $288 - 96t = 0$ •3 $\therefore \ t = 3$ •4 $h(3) = 288(3) - 48(3^2) = 432$ ft (pupils may use mid-point of roots to find max. height) (b) •1 $h'(2\cdot7) = 288 - 96(2\cdot7) = 28\cdot8$ ft/s •2 No since $28\cdot8 < 48$ ft/s (c) •1 $288 - 96t = 48$ •2 $t = 2\cdot5$ sec. •3 $h(2\cdot5) = 720 - 300 = 420$ ft
3.	**ans:** $\{194\cdot5^o \ , \ 345\cdot5^o\}$ **6 marks** •1 double angle substitution •2 simplifying to standard quad. form •3 factorising and solving •4 discarding -2 solution •5 for 1st angle •6 for 2nd angle	•1 $9\sin x + 4 = 2(1 - 2\sin^2 x)$ •2 $4\sin^2 x + 9\sin x + 2 = 0$ •3 $(4\sin x + 1)(\sin x + 2) = 0$ $\therefore \ \sin x = -\frac{1}{4}$ *or* ~~$\sin x = -2$~~ •4 $\sin x = -\frac{1}{4}$ … stated or implied •5 $x = 180 + 14\cdot5 = 194\cdot5$ •6 $x = 360 - 14\cdot5 = 345\cdot5$

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4.	**(a) ans:** P(4,6) **4 marks** •1 startegy + substituting •2 simplifying to quadratic equation •3 factorising + first coordinate •4 second coordinate **(b) ans:** $(x-14)^2+(y-16)^2=20$ **5 marks** •1 stepping out strategy •2 finding original centre •3 establishing the new centre •4 calculating radius *(may use pyth.)* •5 substituting in general equ. to answer	(a) •1 $(2y-8)^2+y^2-4(2y-8)-20y+84=0$ •2 $5y^2-60y+180=0$ •3 $5(y-6)(y-6)=0 \quad \therefore \quad y=6$ •4 $x=2(6)-8=4$ (b) •1 From P to Q …. 12 along , 6 up (or equivalent strategy) •2 $C_1(2,10)$ •3 $C_2(2+12,10+6) = C_2(14,16)$ •4 $r=\sqrt{(-4)^2+(-10)^2-84}=\sqrt{20}$ •5 $(x-14)^2+(y-16)^2=20$
5.	**(a) ans:** proof **2 marks** •1 for knowing to substitute •2 for simplifying to answer **(b) ans:** $U_2=a^3-2a^2+a+1$ **2 marks** •1 knowing to sub (a) into U_2 = … •2 answer **(c) ans:** $a=4$, quotient has no roots (or equivalent) **4 marks** •1 strategy (synthetic division) •2 finding answer for *a* •3 checking for further roots •4 explanation for no further roots	(a) •1 $U_1=a(a-2)+1$ •2 $U_1=a^2-2a+1$ (b) •1 $U_2=aU_1+b$ $U_2=a(a^2-2a+1)+1$ •2 $U_2=a^3-2a^2+a+1$ (c) •1 a | 1 -2 1 -36 •2 4 | 1 -2 1 -36 4 8 36 1 2 9 0 $\therefore a=4$ •3 for x^2+2x+9...... $b^2-4ac=-32$ •4 since $b^2-4ac<0$, no further roots
6.	**(a) ans:** Area = $1\frac{1}{3}$ m^2 **7 marks** •1 for setting up integral •2 integrating •3 substituting in limits •4 calculating area •5 finding *y* coordinate at $x=2$ •6 calculating area of rectangle •7 subtracting to work out shaded area **(b) ans:** 32 m^3 **2 marks** •1 for knowing how to calculate volume •2 for calculations to answer	(a) •1 $A=\int_2^4 (5+2x-\frac{1}{4}x^2)\,dx$ •2 $=\left[5x+x^2-\frac{1}{12}x^3\right]_2^4$ •3 $=(20+16-5\frac{1}{3})-(10+4-\frac{2}{3})$ •4 $=17\frac{1}{3}$ square metres •5 $y=5+2(2)-\frac{1}{4}(2^2)=8$ •6 $A_{rec}=8\times 2=16$ square metres •7 $A_{sh}=17\frac{1}{3}-16=1\frac{1}{3}$ sq. m (b) •1 $V=$ *face area* $\times$ *depth* •2 $V=1\frac{1}{3}\times 6=8$ .... $V_{tot}=8\times 4=32$ m^3

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7.	**(a) ans:** $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$ **3 marks** •1 drawing a R.A. triangle •2 calculating hypotenuse •3 lifting answers **(b) ans:** proof **5 marks** •1 expanding •2 putting in all exact values •3 simplifying •4 rationalising the denominator •5 taking out common factor to answer	(a) •1 drawing triangle •2 $h^2 = 2+4=6 \therefore h=\sqrt{6}$ •3 $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$ (b) •1 $\sin(\theta+\frac{\pi}{3}) = \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}$ •2 $= \frac{2}{\sqrt{6}}(\frac{1}{2}) + \frac{\sqrt{2}}{\sqrt{6}}(\frac{\sqrt{3}}{2})$ •3 $= \frac{1}{\sqrt{6}} + \frac{1}{2}$ •4 $= \frac{\sqrt{6}}{6} + \frac{1}{2}$ •5 $\sin(\theta+\frac{\pi}{3}) = \frac{1}{6}(\sqrt{6}+3)$
8.	**(a) ans:** $y = 3x^2 - x^3$ **3 marks** •1 dealing with the composite function •2 simplifying the composite function •3 subtracting $h(x)$ to answer **(b) ans:** (2,4) **4 marks** •1 knowing to differentiate and solve to 0 •2 finding the two x values •3 finding corresponding y values •4 justifying $x=2$ gives max. $\therefore$ (2,4)	(a) •1 $g(f(x)) = 3(x-1)^2 - 3$ •2 $g(f(x)) = 3x^2 - 6x$ •3 $y = 3x^2 - 6x - (x^3 - 6x) = 3x^2 - x^3$ (b) •1 $\frac{dy}{dx} = 6x - 3x^2 = 0$ •2 $3x(2-x)=0 \quad \therefore \quad x=0 \; or \; x=2$ •3 (0,0) , $y = 3(2^2) - 2^3 = 4 \;\therefore$ (2,4) •4 justification table (or 2nd deriv.)
9.	**(a) ans:** proof **2 marks** •1 expanding and taking to one side •2 removing common factor to required ans. **(b) ans:** a must equal c ($a=c$) **4 marks** •1 condition for equal roots stated or implied •2 drawing out a,b & c and sub. in discrim. •3 simplifying to perfect square •4 conclusion	(a) •1 $ax^2 - ax - cx + c = 0$ •2 $ax^2 - (a+c)x + c = 0$ (b) •1 for equal roots … $b^2 - 4ac = 0$ •2 $a=a$, $b=-(a+c)$, $c=c$ $(a+c)^2 - 4ac = 0$ •3 $a^2 - 2ac - c^2 = (a-c)^2 = 0$ •4 For $(a-c)^2 = 0$ then $a=c$

Total 70 marks