St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 4

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

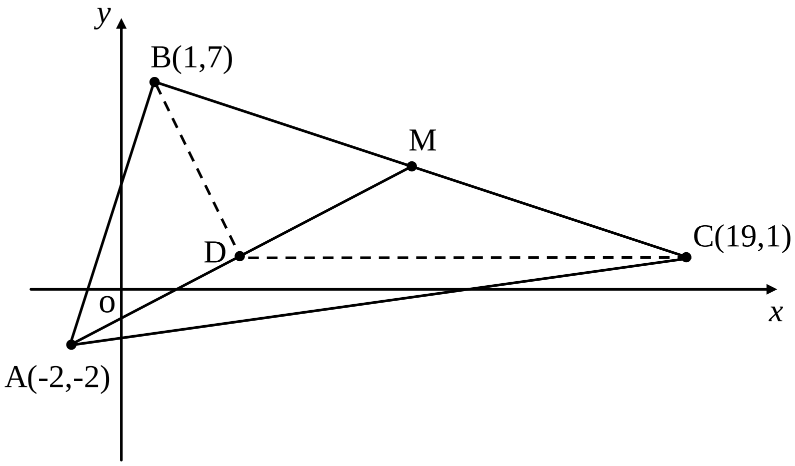
Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. Triangle ABC has vertices (-2,-2), (1,7) and (19,1) as shown.

M is the mid-point of side BC.

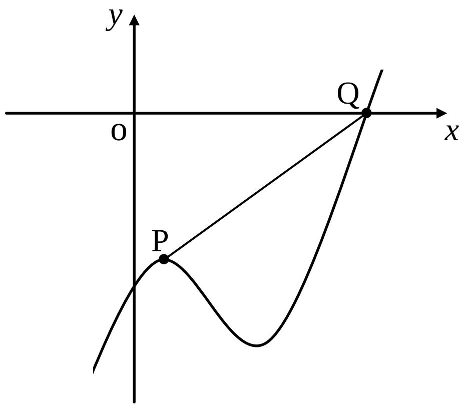


(a) Establish the equation of the median AM. **3**

(b) The **horizontal** line through C intersects AM at D.
Find the coordinates of D. **3**

(c) Hence show clearly that BD is perpendicular to AM. **3**

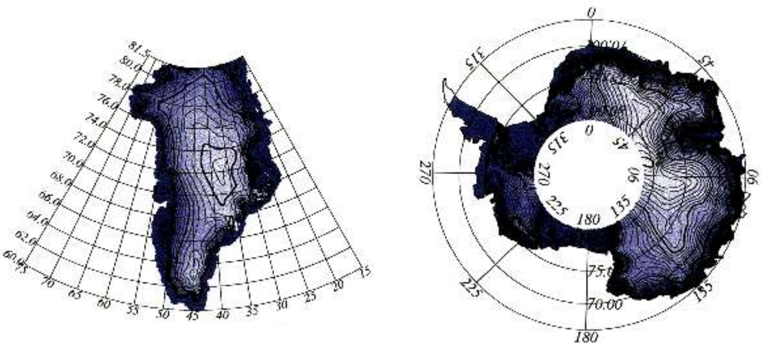
2. Part of the graph of the curve with equation $y = x^3 - \frac{15}{2}x^2 + 12x - 18$ is shown below.
The graph is not drawn to scale.



(a) Find the coordinates of the stationary point P. **4**

(b) Find the coordinates of Q. **3**

3. A scientist is running a computer simulation to represent the possible shrinkage of a small polar ice sheet due to global warming.



He discovers that for this particular simulation the ice sheet is losing 4% of its mass every **2 months**.

(a) Calculate the mass of ice remaining after **10 months** if the initial mass of the simulated ice sheet is 40 gigatonnes (approximately 10 cubic miles of ice). Give your answer correct to 3 significant figures. **3**

(b) For the remaining 2 months of the year (the coldest period) there is no mass loss. During this period the ice sheet **gains** 3·8 gigatonnes of mass due to significant snowfall and the partial freezing of the surrounding sea water.

This yearly cycle is then repeated.

By considering an appropriate recurrence relation, calculate the mass of ice remaining after a 3 year period. **3**

(c) The scientist knows that **over the long term** the mass of the ice sheet will always lie between an **upper and lower limit**.

Calculate these two limits.

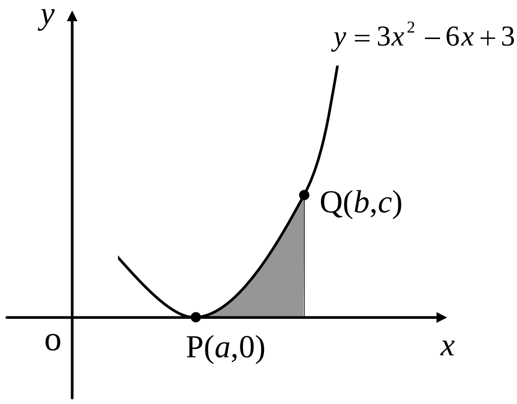
Your answer must be accompanied by appropriate working. **3**

4. Two functions are defined on a suitable domains as $f(x) = x^2 + a$ and $g(x) = x + 1$, where a is a constant.

(a) Find the value of a given that $f\left(g(-2)\right) = -1$ **2**

(b) Hence solve the equation $f\left(f(x)\right) = 2$ **5**

5. The diagram below, which is not drawn to scale , shows part of the graph of the curve with equation $y = 3x^2 - 6x + 3$. The points P(a,0) and Q(b,c) lie on this curve as shown.



(a) Establish the value of a. **2**

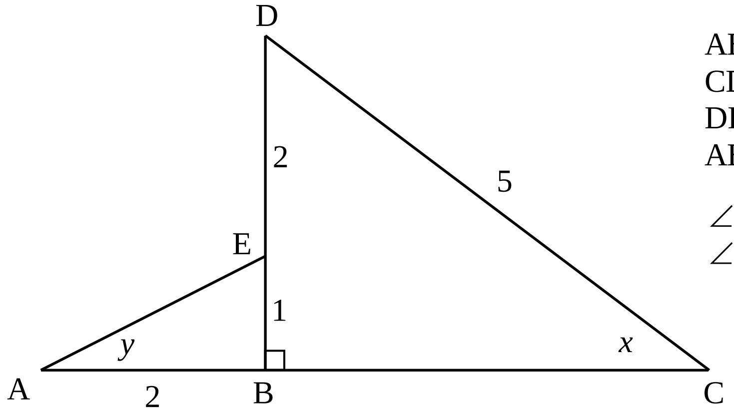
(b) The shaded area (A) can be represented by the integral

$$A = \int_a^b (3x^2 - 6x + 3)\ dx$$

If the shaded area is exactly 1 square unit, find the value of b. **5**

(c) Hence find the equation of the tangent to the curve at Q. **4**

7. Consider the diagram below.



AB = 2 units
CD = 5 units
DE = 2 units
AB = 1 unit

$\angle BCD = x$
$\angle BAE = y$

(a) Calculate the lengths of AE and BC. **2**

8. An amateur rockateer has built a rocket which he hopes will reach a height of at least 4000 feet when using his own home made liquid fuel.

He has modelled the height reached to the mass of fuel used by the formula

$$H(m) = 4m - \frac{m^2}{1200},$$

where H is the height reached in **feet** and m is the mass of fuel used in **millilitres** (ml).

(a) Find the mass of fuel he should use to propel his rocket to its **maximum** height. **4**

(b) What is the predicted maximum height for this rocket when m takes this value? **1**

[**END OF QUESTION PAPER**]