St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 7

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

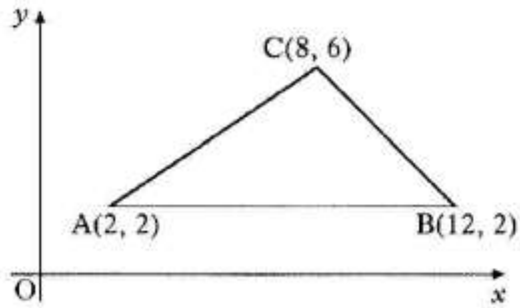
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

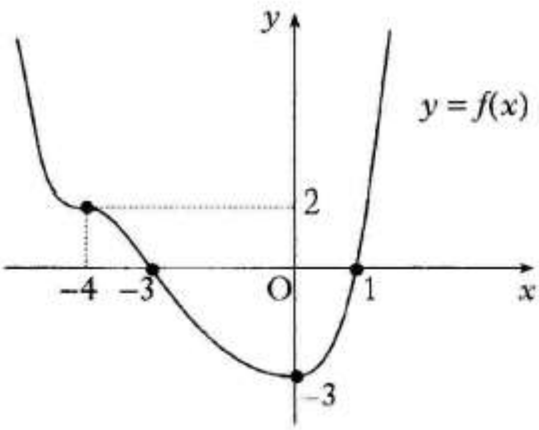
ALL questions should be attempted except those marked as Unit 3

1. Triangle ABC has vertices as shown
   a. Find the perpendicular bisectors of AB and AC.
   b. Hence, find the equation of the circle passing through A, B and C.

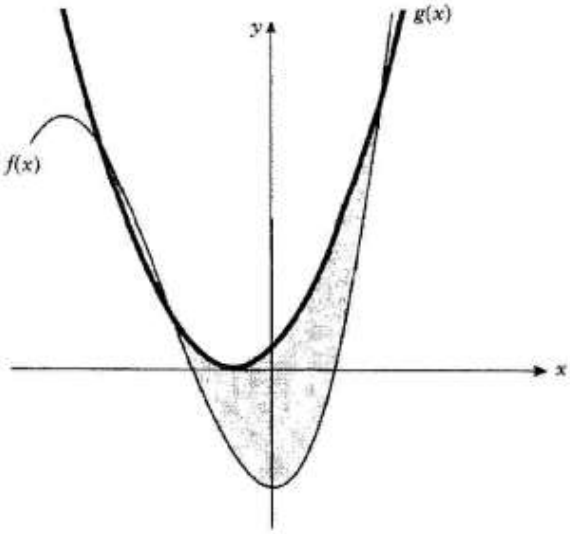


2.
   a. Write $x^2 - 10x + 27$ in the form $(x+b)^2 + c$.
   b. Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.
   c. Use differentiation to establish the value of x that gives the minimum gradient.

3.
   a. State the coordinates of the four points on the graph for
      I. $y = 3f(x)$.
      II. $y = f(3x)$.
   b. Sketch the graph of $y = 3 - f(x)$.



4. The graph shows the functions
   $f(x) = x^3 + 5x^2 - 10$ and
   $g(x) = 2x^2 + 4x + 2$.
   a. Show that $f(-3) = g(-3)$.
   b. Hence find all three points where the graphs intersect.
   c. Calculate the grey area trapped between the curves.



5.

a. Write $3x^2 - 8x + 11$ in the form $(x+b)^2 + c$.

b. Hence, write down the coordinates of the turning point of the parabola with equation $y = 3x^2 - 8x + 11$.

8. In a controlled experiment the number of caterpillars and birds present in a garden were closely monitored. It was decided that the number of birds present was related to the number of caterpillars present on the previous day. Further, it was noticed that the number of caterpillars was related to the number of birds present the previous day.

Let C_n be the number of caterpillars present on day n.

Let B_n be the number of birds present on day n.

The situation has been modelled by the following equations

$$C_{n+1} = 0.1B_n + 100$$
$$B_{n+1} = 2C_n - 100$$

a) Initially there were 200 caterpillars and no birds. Calculate how the situation changed over 5 days.

b) Write down an equation relating C_n to B_{n-1}

c) Write down an equation relating B_{n+1} to B_{n-1}

d) In the long term what does this model predict will happen to the ratio of caterpillars to birds on any particular day?