
Mathematics
Higher Mini-Prelim 1

**NATIONAL
QUALIFICATIONS**

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

- 1. Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** or **D**. Indicate the correct answer by writing **A**, **B**, **C** or **D** opposite the number of the question on your answer paper.

Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. If k is a constant of integration then $\int \sqrt{4x+1} \, dx$ is
- A** $2(4x+1)^{-\frac{1}{2}} + k$
- B** $\frac{2}{3}(4x+1)^{\frac{3}{2}} + k$
- C** $\frac{1}{4}(4x+1)^{\frac{3}{2}} + k$
- D** $\frac{1}{6}(4x+1)^{\frac{3}{2}} + k$
2. If $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 8\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$. The value of $\mathbf{a} \cdot \mathbf{b}$ is
- A** 4
- B** 0
- C** -4
- D** unknown without further information
3. The graph of $y = \log_2 4x$ crosses the x -axis at the point where x equals
- A** 2
- B** 0.25
- C** 4
- D** 0.5
4. Given that $x+2$ is a factor of $x^3 - 2x^2 - 3x + c$, then the value of c is
- A** 10
- B** -10
- C** 6
- D** -6
5. Given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = 4$, the value of $(2\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$ is
- A** 3
- B** -1
- C** 5
- D** 0

6. Given that $f(x) = \frac{1}{(2x-5)^3}$, then $f'(x)$ equals

A $\frac{-6}{(2x-5)^2}$

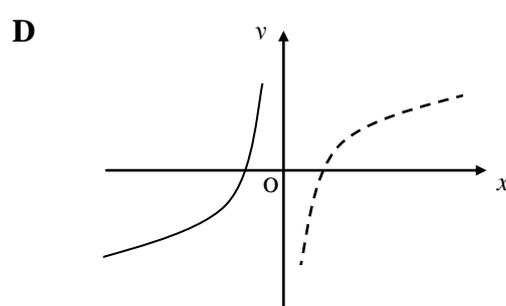
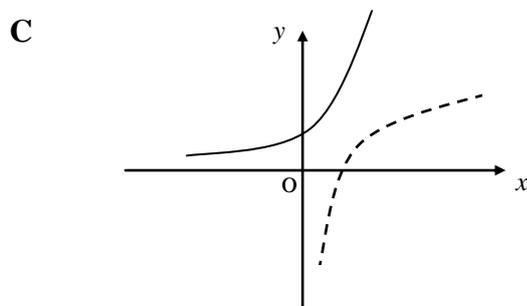
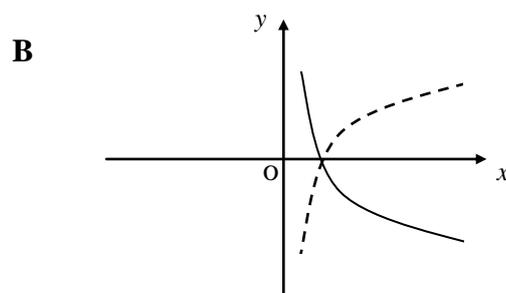
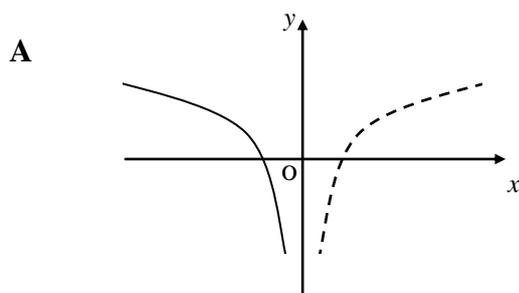
B $\frac{1}{6(2x-5)^4}$

C $\frac{-6}{(2x-5)^4}$

D $\frac{-3}{(2x-5)^4}$

7. Part of the graph of $y = \log_{10} x$ is shown in each diagram below as a broken line.

Which diagram is most likely to show as an unbroken line part of the graph of $y = \log_{10} \frac{1}{x}$?



8. $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$ is equal to

A 0

B -1

C $\frac{1}{2}$

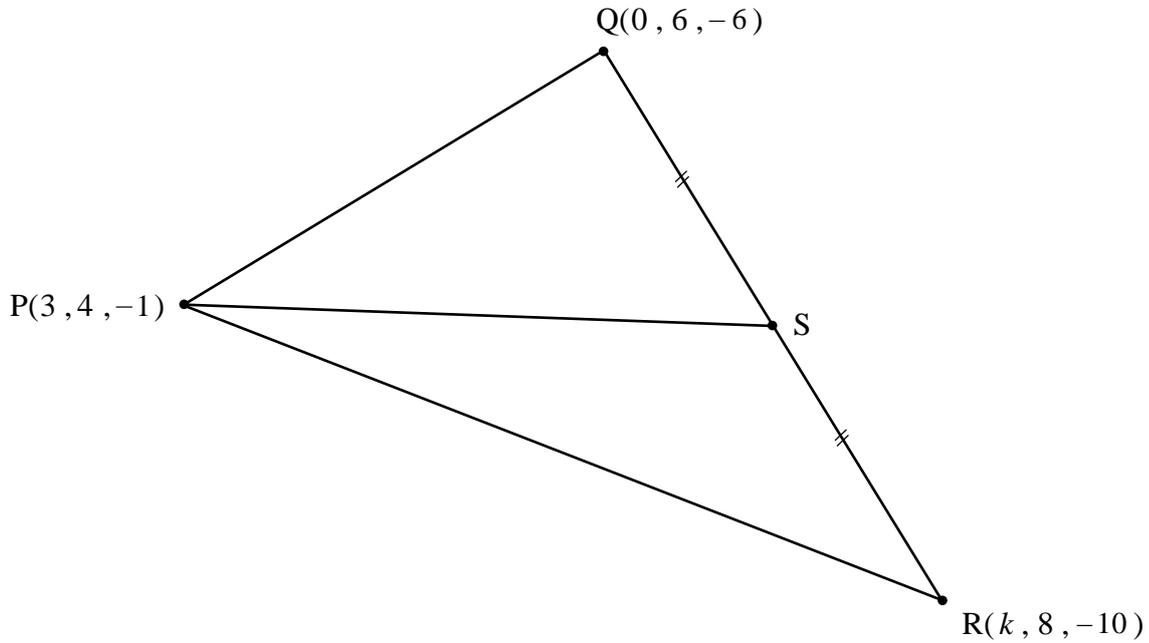
D 1

[END OF SECTION A]

SECTION B

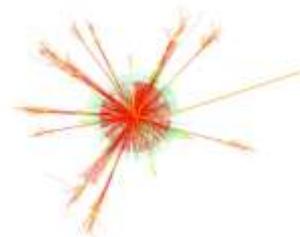
ALL questions should be attempted

9. In the diagram P, Q, and R have coordinates P(3, 4, -1), Q(0, 6, -6) and R(k, 8, -10) respectively.



- (a) Given that angle PQR is a right-angle, find the value of k . **4**
- (b) Calculate the size of angle RPS where S is the mid-point of QR. **6**

10. A Baryon particle decays according to the formula $M_t = M_o e^{-0.0009t}$, where M_o is the initial mass of the substance and M_t is the mass remaining after t seconds.



Calculate, to the nearest ten seconds, how long a sample would take to lose 30% of its original mass.

5

11. (a) Express $3 \cos x^\circ + \sqrt{7} \sin x^\circ$ in the form $k \sin(x+a)^\circ$, where k and a are constants and $k > 0$. 5

(b) Hence state the **minimum** value of f given that $f(x) = \frac{20}{3 \cos x^\circ + \sqrt{7} \sin x^\circ}$. 1

12. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 8$, where a is a constant.

(a) Given that $U_0 = 16$, show that, in terms of a , $U_2 = 8(2a^2 + a + 1)$. 2

(b) Hence find a , where $a > 0$, given that $U_2 = 11$. 3

13. A function is defined on a suitable domain as $h(x) = 2 \sin 2x - \sqrt{3} \cos^2 x$.

Calculate the rate of change of this function at the point where $x = \frac{\pi}{3}$. 5

14. Given that $\log_3(x+1) + 2 \log_3 2 = 2$, find the value of x . 3

[END OF SECTION B]

[END OF QUESTION PAPER]