

# MATHEMATICS

## Higher Grade Extended Unit Test - UNIT 3

Time allowed - 50 minutes

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Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This Unit Test contains questions graded at all levels.**

## FORMULAE LIST

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

**Trigonometric formulae:**

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

### Section A

*In this section the correct answer to each question is given by one of the alternatives A, B, C or D.*

*Indicate the correct answer by writing A, B, C or D opposite the number of the question.*

*Rough working may be done on the paper provided. 2 marks will be given for each correct answer.*

1. Given that  $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$ , which of the following is/are correct?

- (i)  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular
- (ii) the magnitude of  $\mathbf{b}$  is  $4\sqrt{3}$
- (iii)  $\mathbf{a}$  is a unit vector

- A. (i) only
- B. (ii) only
- C. (i) and (ii)
- D. (i) and (iii)

2.  $\frac{d}{dx} \sin(3x - 1)$  is equal to

- A.  $\frac{1}{3} \cos(3x - 1)$
- B.  $3 \cos(3x - 1)$
- C.  $-\frac{1}{3} \cos(3x - 1)$
- D.  $-3 \cos(3x - 1)$

3. If  $f(x) = (4x + x^2)^3$  is, the value of  $f'(1)$  is
- A.  $41\frac{2}{3}$
  - B. 75
  - C. 450
  - D. 1875
4. Given that  $\log_2(x - 1) = \log_3 27$ , the value of  $x$  is
- A. 9
  - B. 7
  - C. 4
  - D. 82
5. The minimum value of  $3 + 5 \cos(x - 53 \cdot 1^\circ)$  is
- A.  $-5$  when  $x = 233 \cdot 1^\circ$
  - B.  $-2$  when  $x = 53 \cdot 1^\circ$
  - C. 3 when  $x = 143 \cdot 1^\circ$
  - D.  $-2$  when  $x = 233 \cdot 1^\circ$

**End of Section A**

**Section B**  
**ALL QUESTIONS SHOULD BE ATTEMPTED**

*In this section credit will be given for all correct working.*

6. A is the point (0, 2, 3), B is the point (2, -2, -2) and C is the point (6, y, -12).
- (a) Given that A, B and C are collinear, calculate the value of  $y$ . 4
- (b) A fourth point, D, divides AC in the ratio 2 : 1.  
Establish the coordinates of D. 3
7. Find  $\int_1^2 (2x - 3)^3 dx$  3
8. Solve for  $x$   $3 \log x + \log 0.75 = \log 6$  3
9. Given that  $f\left(\frac{\pi}{6}\right) = -2$  and  $f'(x) = -8 \sin 4x$ , find an expression for  $f(x)$ . 4
10. Given that  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 26$ ,  $|\mathbf{a}| = 4$  and  $|\mathbf{b}| = 5$ , calculate the size of the angle between  $\mathbf{a}$  and  $\mathbf{b}$ . 3

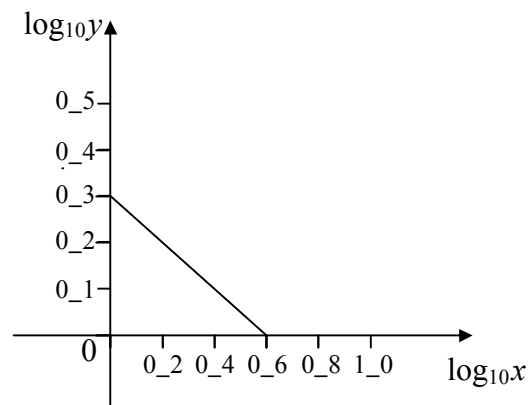
11. (a) Express  $\sin 2x + \sqrt{3} \cos 2x$  in the form  $k \cos(2x - \alpha)$  where  $k > 0$  and  $0 \leq x \leq \frac{\pi}{2}$ . 4

(b) Hence find the value of  $x$  in the interval  $0 \leq x \leq \frac{\pi}{2}$  for which

$$\sin 2x + \sqrt{3} \cos 2x = \sqrt{3} \quad 2$$

12. The graph illustrates the law  $y = kx^n$ .

Find the values of  $k$  and  $n$ .



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**END OF QUESTION PAPER**