

MATHEMATICS

Higher Grade Extended Unit Test - UNIT 3

Time allowed - 50 minutes

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This Unit Test contains questions graded at all levels.**

FORMULAE LIST

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \quad \text{where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

Section A

In this section the correct answer to each question is given by one of the alternatives A, B, C or D. Indicate the correct answer by writing A, B, C or D opposite the number of the question. Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$,

which of the following is/are correct?

- (i) \mathbf{a} and \mathbf{b} are perpendicular
- (ii) The magnitude of \mathbf{b} is $4\sqrt{3}$
- (iii) \mathbf{a} is a unit vector

- A (i) only
- B (ii) only
- C (i) and (ii)
- D (i) and (iii)

2. $\int \sin(3x - 1) dx$ is equal to

- A $\frac{1}{3} \cos(3x - 1) + C$
- B $-\frac{1}{3} \cos(3x - 1) + C$
- C $3 \cos(3x - 1) + C$
- D $-\cos(3x - 1) + C$

3. If $f(x) = (4x + x^2)^3$, the value of $f'(1)$ is

- A $41\frac{2}{3}$
- B 75
- C 450
- D 1875

4. Given that $\log_2(x - 1) = \log_3 27$, the value of x is

- A 9
- B 7
- C 4
- D 82

5. The minimum value of

$$3 + 5 \cos(x - 53 \cdot 1^\circ)$$
 is

- A -5 when $x = 233 \cdot 1^\circ$
- B -2 when $x = 53 \cdot 1^\circ$
- C 3 when $x = 143 \cdot 1^\circ$
- D -2 when $x = 233 \cdot 1^\circ$

Section B

ALL QUESTIONS SHOULD BE ATTEMPTED

In this section credit will be given for all correct working.

6. A is the point (2, -4, 4), B is the point (1, -2, 5) and C is the point (-1, 2, 7).
- (a) Show that A, B and C are collinear. 3
- (b) Write down the ratio in which B divides AC. 1
7. Find $\int \frac{1}{\sqrt[3]{2x+1}} dx$ 3
8. (a) Express $\sqrt{5} \cos 2x^\circ - 2 \sin 2x^\circ$ in the form $k \sin(2x - \alpha)^\circ$ where k and α are constants and $k > 0$. 4
- (b) Hence solve the equation $3 + \sqrt{5} \cos 2x^\circ - 2 \sin 2x^\circ = 2$, where $180 < x < 360$ 4
9. L, M and N are the points (2, 0, -1), (4, -6, 8) and (-5, 7, 0) respectively.
- Calculate the size of angle LMN. 5
10. Find the gradient of the tangent to the curve with equation $y = 2 \cos 3x - \sin^2 x$ at the point with x -coordinate $\frac{\pi}{2}$. 4

11. A radioactive substance decays according to the formula $M_t = M_o e^{-0.004t}$ where M_o is the initial mass of the substance and M_t is the mass remaining after t days.
- (a) Show that the time taken for the substance to lose half its mass can be written as $t = 250 \log_e 2$. 4
- (b) If the initial mass was 500g, calculate the mass remaining after 500 days. 2

END OF QUESTION PAPER